UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Astronomy 4C17: Galaxy and Cluster Dynamics

COURSE CODE	:	**ASTR4C17**
UNIT VALUE	:	**0.50**
DATE	:	**05-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

06-C0095-3-40

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Solar radius	$R_\odot$	=	6.96×10^{8} m
Solar mass	$M_\odot$	=	2.0×10^{30} kg
Solar Luminosity	$L_\odot$	=	3.83×10^{26} J s^{-1}
Parsec	pc	=	3.09×10^{16} m
Gravitational constant	G	=	6.67×10^{-11} N m^2 kg^{-2}
Distance to Galactic Centre	R_0	=	10 kpc
Sun's orbital circular velocity	v_0	=	250 kms^{-1}
Oort's constants	A	=	15 kms^{-1} kpc^{-1}
	B	=	−10 kms^{-1} kpc^{-1}

$$\int_{\sqrt{6}}^{\infty} x^2 e^{-x^2} dx \simeq 0.00328$$

The 1-d Poisson Equation in spherical geometry is:

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)$$

$$A + B = -\left(\frac{dv_c}{dr}\right)_{R_0}, A - B = \frac{v_0}{R_0}$$

$$\int_{-\infty}^{\infty} x e^{-\beta^2 x^2} dx = 0$$

$$\int_{-\infty}^{\infty} x^2 e^{-\alpha^2 x^2} dx = \frac{\pi^{\frac{1}{2}}}{2\alpha^3}$$

1.(a) What does the distribution of stars in phase space $f(\underline{r}, \underline{v}, t)$ physically describe? Use Newton's Second Law of Motion to show that the Collisionless Boltzmann Equation of Stellar Dynamics is given by :-

$$0 = \frac{\partial f}{\partial t} + v.(\nabla_r f) - (\nabla_r \phi_p).(\nabla_v f)$$

where ϕ_p is the gravitational potential of the galaxy and the subscripts r and v refer to derivatives with respect to position and velocity respectively. You should state all the assumptions and simplifications used in this derivation. **[10]**

(b) A simple ('isothermal sphere') model of *spherical* globular clusters assumes $f = ke^{-BE}$, where E is the total energy of a star ($E = v^2/2 + \Phi_{\mathrm{p}}(r)$), and k and B are constants. Show that the number density of stars is given by:

$$n(r) = k\left(\frac{2\pi}{B}\right)^{3/2} e^{-B\Phi_{\mathrm{p}}(r)}$$

[3]

Using this result calculate $n(r)$, in terms of k and r, for (i) $\Phi_p(r) = r$ and (ii) $\Phi_p(r) = (\ln r)/B$. For each function of $\Phi_p(r)$ calculate the radius at which $n(r)$ is a minimum. Comment on the density distribution. **[3]**

(c) Show (by integrating) that for a distribution function of the form

$$f = A\ exp\left[-(\alpha^2 u^2 + \beta^2 (v - v_c)^2 + \gamma^2 w^2)\right]$$

where A, α, β and γ are constants, the mean velocity S has $(\overline{u}, \overline{v}, \overline{w})$ components given by $(0, v_c, 0)$.

Show that if v_c is zero (giving $f = A\ exp[-(\alpha^2 u^2 + \beta^2 v^2 + \gamma^2 w^2)]$), then the expected value of S^2 is given by

$$\langle S^2 \rangle = \frac{1}{2}\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}\right)$$

[4]

2.(a) Derive the general form of the virial equation for a relaxed system composed of N point masses. **[6]**

How is this equation modified if the system is in equilibrium? What are the main properties of a virial system in equilibrium? Derive the virial mass of such a system. [4]

(b) Use the virial theorem to show that for a cluster

$$\frac{dR}{R} = \frac{2dM}{M} - \frac{dE}{E}$$

[2]

Show that, if the cluster evolves primarily as a result of stars driving winds that are *just* ejected from such a cluster, then,

$$R \propto \frac{1}{M}.$$

You may assume that the cluster remains well-virialised, so that the escape velocity ($v_{esc.}$) is related to the stellar r.m.s. speed (v) by $\langle v_{esc.}^2 \rangle = 4\langle v^2 \rangle$. [3]

(c) Discuss qualitatively how a cluster loses stars via *evaporation* [2]

Assuming that the velocity distribution in a (spherical) globular cluster is given by the normalised Maxwellian,

$$f(v) = \frac{1}{(2\pi)^{3/2}\sigma^3} e^{-v^2/2\sigma^2}, \qquad \sigma = \sqrt{\langle v^2 \rangle / 3}.$$

Calculate the proportion of stars that are lost 'per relaxation period'. You may use the numerical result given in the preamble. [3]

3.(a) Define the terms *kinematic* and *dynamical local standards of rest* in the Galaxy. What is the *Stromberg asymmetric drift*? Explain (in simple terms) its physical origin. [8]

(b) Evaluate the radial dependence of the circular speed of stars, $v_c(r)$, in galaxies whose mass distribution is represented by (i) a homogeneous sphere, (ii) an inhomogeneous sphere in which $\rho = kr^{-2}$ and (iii) an inhomogeneous sphere in which $\rho = k\,e^{[-r]}/r$. [6]

(c)In the Schmidt model, the rotational velocity in the Galactic plane (z=0) is given by:

$$v_c^2(r) = 4\pi G\sqrt{1-e^2}\int_0^r \frac{\rho(a)a^2da}{\sqrt{r^2-a^2e^2}}$$

where e is the eccentricity and $\rho(a)$ is an arbitrary density distribution. When this model was applied to a distant young galaxy it was found that the density distribution could be approximated by

$$\rho(a) = P_{-1}/a,$$

where P_{-1} is a constant. Use the above to show that $v_c^2(r) = 3.587 \times 10^{-11} P_{-1} r$. You may assume that $e = 0.999$. [Hint use a $sin\theta$ substitution].
Then use the definitions of the Oort constants, and assuming that $e = 0.999$, to obtain values for P_{-1}. Which value of P_{-1} is physical and why? Hence state an expression for $\rho(a)$ (in S.I. units) for r =10kpc. [6]

4.(a) Discuss what is meant by *phase damping* and *violent relaxation.* [7]

Show *qualitatively* why a galaxy that has recently undergone violent relaxation looks different to one that has relaxed over a long period of time by two-body relaxation alone? [3]

Comment on the applicability of violent relaxtion to E, S and S0 type galaxies. [1]

(b) In a numerical simulation to test the effects of violent relaxation on the stellar dynamics of an elliptical galaxy of mass M a test particle's mean orbital radius was found to be increasing with time in the following way :

$$r(t) = R_o \left(1 + \frac{t}{\tau_c}\right)^{\frac{1}{4}},$$

where R_o is the mean orbital radius at $t = 0$ and τ_c is a constant. Write down the gravitational potential ϕ_p of this test particle, assuming the galaxy can be treated as a point mass, and thus derive expressions showing (i) how the energy E of this particle changes with time (dE/dt) and (ii) how the radial velocity of this particle changes with time ($dr(t)/dt$) . [4]

(c) If $\tau_c = 10^7$ years, $R_o = 10^4$pc and the mass of the galaxy is $M = 10^{11} M_\odot$ sketch (i) the radius, (ii) the radial velocity and (iii) the gravitational potential of the test particle at times between 10^3 and 10^9 years. [3]

At what time will the change in the potential energy of the test particle since $t = 0$ be equal to the total energy of the particle at $t = 0$? [You may assume the system was originally in virial equilibrium]. [2]

5.(a) Using the Oort model of Galactic rotation show that the radial and tangential velocities of a star are given by $v_{rad} = A\,d\sin 2l$ and $v_{tan} = A\,d\cos 2l + Bd$ respectively. State all assumptions and define all terms. [10]

Note that

$$\left(\frac{d\dot{\theta}}{dr}\right)_{R_o} = \frac{1}{R_o}\left(\frac{d\Theta}{dr}\right)_{R_o} - \frac{\Theta_o}{R_o^2}.$$

(b) Use the virial theorem to show that the rate at which a galactic cluster loses energy is related to the rate at which its radius changes by the expression

$$\frac{dE}{dt} = \frac{\alpha}{2}\frac{Gm_{cl}^2}{r_{cl}^2}\frac{dr_{cl}}{dt},$$

where m_{cl} is the mass of the cluster, r_{cl} is the radial size of the cluster and α is a constant. You may assume that the potential energy of a galactic cluster is given by

$$U = -\frac{\alpha G m_{cl}^2}{r_{cl}}.$$

[4]

Thus show that the lifetime against shocks t_{sh} for a cluster that encounters a molecular cloud is given by

$$t_{sh} = \frac{\alpha v_{mc}}{6G}\frac{\rho_{cl}}{\rho_{mc}}\frac{R_{mc}^2}{m_{mc}}$$

You may assume that

$$\rho_{cl} = \frac{m_{cl}}{4\pi r_{cl}^3/3} \quad \text{and that} \quad \frac{dE}{dt} = \frac{4\pi G^2 m_{cl} r_{cl}^2 \rho_{mc} m_{mc}}{3 v_{mc} R_{mc}^2}.$$

You must define all terms and state all assumptions. [6]