Solution 4(a)

The frequency of the gravitational waves from the binary is equal to

$$\omega = 2\omega_0 = 2\sqrt{\frac{GM}{r^3}},$$

hence

$$r = \left(\frac{GM}{\omega_0^2}\right)^{1/3} = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3}$$

and the amplitude of the gravitational waves

$$h_0 = \frac{\alpha r_g^2}{2rR} = \frac{\alpha r_g^2}{2R}\left(\frac{4\pi^2}{GMT^2}\right)^{1/3}.$$

Let us assume that some future detector of gravitational waves, for example LISA, will be able to detect gravitational waves with

$$h_0 > h_*,$$

Hence, the distance, R, should satisfy to the inequality

$$R < R_{max} = \frac{\alpha r_g^2}{2h_*}\left(\frac{4\pi^2}{GMT^2}\right)^{1/3} = Ah_*^{-1}M^{5/3}T^{-2/3},$$

where

$$A = \frac{4\alpha G^2}{2c^4}\left(\frac{4\pi^2}{G}\right)^{1/3} = 2^{5/3}\alpha G^{5/3}\pi^{2/3}c^{-4}.$$