Solution 3(c)

If r>rlso mater spirals slowly and kinetic energy is negligible, hence gravitational energy transforms to the luminosity of the disk, while if r<rlso mater falling nearly radially and gravitational energy transforms predominantly to the kinetic energy and luminosity of this region of the disk is negligible.

We have

$$l(x) = \frac{3}{8\pi}\frac{GM\dot{m}}{r_{\text{lso}}^3} x^6 (1-x); \qquad x = \left(\frac{r}{r_{\text{lso}}}\right)^{-1/2}.$$

so that

$$\frac{\partial l(x)}{\partial x} = \left[\frac{3}{8\pi}\frac{GM\dot{m}}{r_{\text{lso}}^3}\right]\left[6x^5(1-x) - x^6\right] = 0,$$

for a maximum, giving

$$x = \frac{6}{7} \quad or \qquad r = \left(\frac{7}{6}\right)^2 r_{\text{lso}} \approx 1.36\, r_{\text{lso}},$$

so that

$$l_{\max} = \frac{6^6}{7^7}\frac{3}{8\pi}\frac{GM\dot{m}}{r_{\text{lso}}^3} = \frac{6^6}{7^7}\frac{3}{8\pi}\frac{GM\dot{m}}{\left(6GM/c^2\right)^3} = \frac{6^3}{7^7}\frac{3}{8\pi}\frac{\dot{m}c^6}{(GM)^2}$$

If we assume that the discs radiates as a black body, then

$$l_{\max} = \sigma T_{\max}^4$$

so that

$$T_{\max} = \left[\frac{6^3}{7^7}\frac{3}{8\pi}\frac{\dot{m}c^6}{\sigma (GM)^2}\right]^{1/4}$$

Putting in numbers, we get

$$T_{\max}(K) = \left[\frac{6^3}{7^7}\frac{3}{8\pi}\frac{\left(2\times10^{30}/3.14\times10^7\right)\left(3\times10^8\right)^6}{5.67\times10^{-8}\times\left(6.67\times10^{-11}\times2\times10^{38}\right)^2}\right]^{1/4}$$

$$\approx 10^5.$$