

SOLUTION 3(B)

Photons are massless and each carries momentum \tilde{p} given by

$$\tilde{p} = E/c$$

and the total momentum \tilde{P} carried per unit time by all the photons emitted by a source of luminosity L is given by

$$\tilde{P} = \frac{L}{c}.$$

The pressure p_γ exerted by these photons is, by definition, the momentum flux, that is the rate flow of momentum per unit area of surface perpendicular to the flow of radiation. At a distance r from the source, therefore,

$$p_\gamma(r) = \left(\frac{L}{4\pi r^2} \right) / c.$$

This pressure acts upon the Thomson cross-sections σ of the (ionised) in-falling particles, mainly protons and electrons. For a particle of charge e and mass m ,

$$\sigma(e, m) = \frac{2}{3} \left(\frac{e^2}{4\pi\epsilon_0 mc^2} \right)^2$$

The main outward force $F_{\text{out}}(r)$ is therefore that exerted by the photons on the electrons and is given by

$$F_{\text{out}}(r) = p_\gamma(r) \times \sigma_T = \frac{L\sigma_T}{4\pi r^2 c},$$

where σ_T is the Thomson cross-section of the electron. Because the electrons in the plasma are coupled to the protons *via* electromagnetic forces, this outward force is transferred to the protons as well.

The gravitational force on the proton is

$$F_{\text{in}}(r) = \frac{GMm_p}{r^2},$$

where m_p is the proton's mass. Again, though, this force is transferred to the electrons electromagnetically.

For accretion to take place, we need inward force to be greater than the outward force given

$$\frac{GMm_p}{r^2} \geq \frac{L\sigma_T}{4\pi r^2 c}.$$

This means that, if a source is to derive its luminosity from accretion, its luminosity L must be less than the *Eddington luminosity*, $L_{\text{Eddington}}$:

$$L \leq L_{\text{Eddington}} := 4\pi \frac{GMm_p c}{\sigma_T}.$$