

## Solution 2(b)

On average, the luminosity of a main sequence star is proportional to its mass to the power 3.3:

$$L(M) = L_* \left( \frac{M}{M_*} \right)^{3.3},$$

where  $L_*$  is the luminosity of a star of mass  $M_*$ . Using the same approach as in 2(a), we get

$$\begin{aligned} L_{\text{total}} &= \int_{M_{\text{low}}}^{M_{\text{high}}} \phi(M) L(M) dM \\ &= \phi_* L_* \int_{M_{\text{low}}}^{M_{\text{high}}} \left( \frac{M}{M_*} \right)^{3.3-\alpha} dM \\ &\approx \frac{\phi_* M_* L_*}{(4.3-\alpha)} \left( \frac{M_{\text{high}}}{M_*} \right)^{4.3-\alpha}, \end{aligned}$$

since  $\alpha < 4.3$ . It is clear that, in contrast to the total mass, the total luminosity is controlled by the (equally uncertain) *upper* mass-limit.