

Solution 2(a)

We have

$$M_{\text{total}} = \int_{M_{\text{low}}}^{M_{\text{high}}} \phi(M) M dM = \phi_* M_* \int_{M_{\text{low}}}^{M_{\text{high}}} \left(\frac{M}{M_*} \right)^{1-\alpha} dM$$
$$\approx \frac{\phi_* M_*^2}{(\alpha - 2)} \left(\frac{M_{\text{low}}}{M_*} \right)^{2-\alpha},$$

where the last line follows because $\alpha > 2$ and $M_{\text{high}} \gg M_{\text{low}}$. The equation shows that the total mass is very sensitive to the uncertain lower-mass limit M_{low} , diverging as M_{low} tends to zero.