

UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Physics 4461: Plasma Physics

COURSE CODE : PHYS4461

UNIT VALUE : 0.50

DATE : 07-MAY-03

TIME : 14.30

TIME ALLOWED : 2 Hours 30 Minutes

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Permittivity of Free Space $\epsilon_0 = 8.8 \times 10^{-12} \text{ F m}^{-1}$

Electronic Charge: $e = 1.6 \times 10^{-19} \text{ C}$

Permeability of Free Space: $\mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$

Proton Rest Mass: $m_p = 1.67 \times 10^{-27} \text{ kg}$

1. A positive point charge Q is slowly inserted into a plasma.

Discuss the effective screening for a

(i) **cold plasma** and (ii) **hot plasma** [4]

Using Boltzmann's equation, derive the equation

$$\nabla^2 \phi = 2\lambda_D^{-2} \phi$$

where ϕ is the plasma potential and λ_D is the Debye length. [8]

A fusion plasma with an effective thermal energy of 1 keV has an electron density of 10^{19} m^{-3} . Determine λ_D . [2]

Discuss the role of λ_D in producing

(i) **coherent** and (ii) **incoherent** oscillations in the plasma. [6]

2. Sketch the geometry of a Tokamak showing the
- (i) Toroidal and Poloidal directions
 - (ii) Major (R_0) and Minor (r_0) radii
 - (iii) Electron path in absence of trapping. [5]

Electron trapping with $V_z = 0$ in an inhomogeneous magnetic field B will occur when

$$B > B_0 V_0^2 (V_{\perp 0})^{-2}$$

where B_0 is the field on the outside of the toroidal surface and V_0 and $V_{\perp 0}$ are the initial total and initial perpendicular velocities respectively. The z direction is the axis of symmetry. [2]

- (i) Give an expression for the dependence of B on R_0 , r_0 , and the poloidal angle θ
- (ii) Show that the toroidal velocity V_z is given by

$$V_z^2 = V_0^2 - V_{\perp 0}^2 (1 - \epsilon \cos \theta)(1 - \epsilon)^{-1}$$

where ϵ is the inverse aspect ratio. [4]

- (iii) Derive the condition for the establishment of **Passing** and **Trapped** electrons. Why is it necessary to apply a Poloidal field in addition to the Toroidal field? [9]

3. In a cold plasma the dielectric tensor ϵ for the propagation of electromagnetic waves of frequency ω and amplitude E is given by

$$\epsilon = I + i(\epsilon_0 \omega)^{-1} \sigma$$

where σ is the conductivity tensor and I is an identity matrix.

- (i) Show that the general wave equation is

$$\nabla^2 E - \nabla(\nabla \cdot E) + \frac{\omega^2}{c^2} \epsilon \cdot E = 0 \quad [4]$$

- (ii) Derive the dispersion relationship for the propagation of transverse modes in the plasma [4]

In a cold magnetic plasma the velocity $V(t)$ of mode propagation is given by

$$V(t) = a \cdot E$$

where E is the electric field and \underline{a} is a 3×3 tensor functionally

dependent on the radiation frequency ω and the electron cyclotron frequency

ω_{ce} . Derive the tensor a . You may assume that $\omega \neq \omega_{ce}$. [12]

4. Consider a perfectly conducting incompressible fluid moving with velocity \mathbf{V} with respect to a laboratory frame. Magnetic and electric fluid exist in the laboratory frame.

(i) Show that the evolution of \mathbf{B} in the fluid is given by

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) \quad [4]$$

(ii) Using the equation of continuity, derive the velocity condition for an incompressible fluid. [4]

(iii) Hence show that $\frac{d\mathbf{B}}{dt} = (\mathbf{B} \cdot \nabla)\mathbf{V}$ [4]

(iv) Consider the fluid element at $(\mathbf{t}, \mathbf{x}_i)$ and subsequently at $(\mathbf{t} + \Delta t, \mathbf{x}_j)$. Derive the Stress Tensor relating \mathbf{B} at i and j . [8]

5. An intense laser field of wavelength λ , angular frequency ω and amplitude E interacts with a

neutral atomic gas producing a plasma:

- (i) Distinguish between Multiphoton ionization and Barrier Supression Ionization. [2]

- (ii) Explain how the Pondermotive Potential, ϕ influences the multi photon ionization process. [2]

- (iii) Derive the expression

$$\phi = \frac{e^2 E_0^2}{4m\omega^2}$$

stating clearly any approximations used. [4]

- (iv) An electron is bound by a Coulomb potential to the nucleus of charge z with an ionization potential V . Show that the threshold laser electric field required to produce Barrier Suppression ionization is

$$E_T = Z^3/16nt$$

where n is the principle quantum number. [4]

- (1) When $E < E_T$ show that the tunnelling probability is proportional to $\exp(-V^{3/2}/E)$.

State clearly any approximations used. [8]

END OF PAPER