UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

M.Sci.

Physics 4442: Particle Physics

COURSE CODE	:	**PHYS4442**
UNIT VALUE	:	**0.50**
DATE	:	**10-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

06-C1184-3-50

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

In this paper you may assume the mass of the W to be 80 GeV, the mass of the Z to be 91 GeV, the quark masses to be: u 1 MeV, d 2 MeV, s 0.2 GeV, c 1.2 GeV, b 4.5 GeV and t 175 GeV; the charged lepton masses to be: e 0.5 MeV, μ 0.1 GeV and τ 1.7 GeV; and the neutrinos to be massless.

Also given: 1 barn = 10^{-28} m^2 and in natural units 1 m = 5.068×10^{15} GeV^{-1}.

[Part marks]

1. The PEP-II accelerator at SLAC collides head-on electrons of energy 9 GeV with positrons of energy 3.1 GeV.

   (a) Calculate the centre-of-mass energy E_{CM} of the collisions. [3]

   (b) The differential cross section for $e^+e^- \to \mu^+\mu^-$ is given by

   $$\frac{d\sigma}{d(\cos\theta)} = \frac{e^4(1+\cos^2\theta)}{32\pi s},$$

   where e is the electric charge of the electron and s the square of the centre-of-mass energy. Draw the lowest order Feynman diagram for this process and justify the fourth power of e in the above expression. [4]

   (c) Integrate the differential cross section to obtain an expression for the total cross section of the above process in terms of s and the fine structure constant $\alpha = e^2/4\pi$. Using $\alpha = 1/137$, calculate the total cross section at the PEP-II centre-of-mass energy in nb. [3]

   (d) The instantaneous luminosity of PEP-II is $\mathcal{L} = 10^{34}$ cm^{-2} s^{-1}. Assuming that the accelerator runs for a total of 180 days in a year, calculate how many muon pair events are produced per year. [3]

   (e) Ignoring any resonance effects, estimate the ratio

   $$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

   Explain whether R would increase or decrease if resonance effects were to be taken into account. R is measured precisely to be 4.5. What do you conclude by comparing this number to your estimate? [7]

2. The Lagrangian density for free electrons can be written as

$$\mathcal{L}_{\mathrm{D}} = \imath\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi\,,$$

where $\overline{\psi} = \psi^{\dagger}\gamma^{0}$ and the γ matrices satisfy $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2g^{\mu\nu}$.

(a) Show that the Euler-Lagrange equations for continuous field variables $\phi_i(x)$

$$\partial_{\mu}\left(\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi_i)}\right) - \frac{\partial\mathcal{L}}{\partial\phi_i} = 0$$

applied on $\mathcal{L}_{\mathrm{D}}$ give the Dirac equation for ψ (taking $\phi_i = \overline{\psi}$) and the adjoint Dirac equation for $\overline{\psi}$ (taking $\phi_i = \psi$). [3]

(b) Show that $\mathcal{L}_{\mathrm{D}}$ is invariant under the global phase transformation

$$\psi \to \psi \mathrm{e}^{-\imath q\lambda}\,,$$

where λ is a continuous variable that does not depend on space-time and q is the charge of the electron. [2]

(c) According to Nöther's theorem, for every symmetry of a Lagrangian density under the change of a continuous variable λ, there is a conserved current J^{μ} given by

$$J^{\mu} = \sum_i \frac{\partial\mathcal{L}}{\partial(\partial_{\mu}\phi_i)}\frac{\partial\phi_i}{\partial\lambda}\,.$$

Derive the conserved current corresponding to the above symmetry in $\mathcal{L}_{\mathrm{D}}$. What is the physical interpretation of it? [5]

(d) Show that $\mathcal{L}_{\mathrm{D}}$ is not invariant under a local phase transformation, i.e. if λ above has a space-time dependence, and find the extra term which arises as a result of this transformation. [4]

(e) Consider the addition to $\mathcal{L}_{\mathrm{D}}$ of an interaction term given by

$$-qA_{\mu}\overline{\psi}\gamma^{\mu}\psi\,,$$

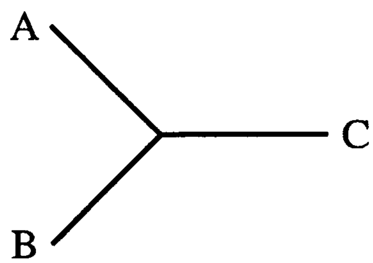
where A_{μ} is the electromagnetic potential. Which transformation of A_{μ} allows $\mathcal{L}_{\mathrm{D}}$ to become invariant under local phase transformations? Explain the significance of this result. [6]

[Part marks]

3. (a) Discuss briefly (not necessarily using maths) the procedure of renormalisation. [4]

(b) Explain qualitatively the running of the electromagnetic coupling constant. What is the connection with the procedure of renormalisation? [4]

(c) An imaginary world consists of just three types of particles: A, B and C. They all have spin 0 and each is its own antiparticle. There is only one vertex by which the particles interact:



and the strength of the interaction is determined by a coupling constant g.

i. Draw the lowest order Feynman diagram(s) and determine the amplitude, $\mathcal{M}$, for the process $A + B \to A + B$. Express $\mathcal{M}$ in terms of the Mandelstam variables. [6]

ii. The differential cross section for a two-body scattering process in the centre-of-mass (CM) frame is given by Fermi's Golden Rule:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|\mathcal{M}|^2}{E_{CM}^2} \frac{|\vec{p}_f|}{|\vec{p}_i|},$$

where $|\vec{p}_i|$ and $|\vec{p}_f|$ are the initial and final state momenta. Using this, and assuming that $m_A = m_B$ and $m_C = 0$, derive an expression for the differential cross section of $A + B \to A + B$ in the CM frame in terms of the CM energy, E_{CM}, and the scattering angle, θ. You may assume that E_{CM} is high enough that approximations such as $E_A \approx |\vec{p}_A|$ can be made. [6]

[Part marks]

4. The Higgs mechanism, with the consequent Higgs boson, is postulated in the Standard Model as the method to give mass to the particles we observe. LEP2 was a circular electron-positron collider at CERN that operated at centre of mass energies, E_{CM}, up to 208 GeV and searched for the Higgs boson.

   (a) Draw the Feynman diagram for the dominant Higgs production mechanism at LEP2. This mechanism sets a kinematic limit on the mass of the Higgs, m_H, which could be produced. At a centre-of-mass energy of 208 GeV, estimate this limit (you may ignore the Z width). For Higgs masses less than this limit, show that the energy of the Higgs produced is given by

   $$E_H = \frac{E_{cm}^2 + m_H^2 - m_Z^2}{2E_{cm}},$$

   where m_Z is the mass of the Z boson. [6]

   (b) How does the coupling of the Higgs to a fermion depend on the fermion's mass? [4]

   (c) Give the main decay mode of the Higgs for a Higgs mass around 115 GeV and estimate its branching fraction, neglecting any differences in phase space of its decay modes. [6]

   (d) At CERN's Large Hadron Collider, a proton-proton collider that will start operation in 2007 at centre-of-mass energy of 14 TeV, the main decay mode that will be used to search for a Higgs of this mass is $H \to \gamma\gamma$. Draw the corresponding Feynman diagram and discuss it briefly in relation to the Standard Model Higgs couplings. Why is the search strategy different from LEP2? [4]

5. The Z boson has both vector and axial couplings to fermions, given by

$$c_{fV} = \pm 1/2 - 2Q_f \sin^2\theta_W, \qquad c_{fA} = \pm 1/2,$$

where Q_f is the fermion charge, θ_W is the weak mixing angle. The positive sign is taken for neutrinos and the u, c and t quarks, and the negative sign is taken for all other fermions.

The LEP collider at CERN produced millions of Z bosons in e^+e^- collisions at centre-of-mass energies near 91 GeV. These Z bosons subsequently decayed into one of several fermion-antifermion pairs.

(a) Draw the lowest order Feynman diagram for the production and decay of a Z boson in the above reaction. List the fermions which the Z can decay into. [5]

(b) Assuming all the relevant fermions can be treated as massless, the partial width for the Z to decay to a fermion-antifermion pair is

$$\Gamma_f = \frac{G_F m_Z^3}{6\pi\sqrt{2}}(c_{fV}^2 + c_{fA}^2),$$

where $G_F = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$ is the Fermi coupling constant and $m_Z = 91.19\,\mathrm{GeV}$ is the Z mass. Evaluate the partial width for the Z to decay "invisibly", meaning to any neutrino-antineutrino pair. [5]

(c) The cross section to any fermion-antifermion pair at a centre of mass energy of m_Z is given by

$$\sigma_f = \frac{12\pi}{m_Z^2\Gamma_Z^2}\Gamma_e\Gamma_f,$$

where $\Gamma_Z = 2.495\,\mathrm{GeV}$ is the total Z width. Calculate the partial width to e^+e^- using the approximation that $\sin^2\theta_W = 1/4$ and hence estimate the total "visible" cross section in nb (1 barn = $10^{-28}\,\mathrm{m}^2$ and in natural units 1 m = $5.068 \times 10^{15}\,\mathrm{GeV}^{-1}$). [5]

(d) The Z width is measured by the LEP experiments to an accuracy of 0.1% and the visible cross section to 2%. Estimate the change which would occur in the Z width and the visible cross section if there were a fourth generation massless neutrino. State an important consequence of this result. [5]

END OF PAPER