UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Physics 4421: Atom and Photon Physics

COURSE CODE	:	**PHYS4421**
UNIT VALUE	:	**0.50**
DATE	:	**13-MAY-04**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

04-C1101-3-40

TURN OVER

Answer any THREE questions

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Electronic charge : $e = 1.6 \times 10^{-19}$ C

Mass of proton : $m_p = 1.67 \times 10^{-27}$ kg

Planck constant : $h = 6.63 \times 10^{-34}$ J s

Speed of light in a vacuum : $c = 3.0 \times 10^{8}$ m s^{-1}

First Bohr orbit : $a_o = 5.29 \times 10^{-11}$ m

Permittivity of free space: $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1}.

Boltzmann constant : $k = 1.38 \times 10^{-23}$ J K^{-1}

[Part Marks]

1. Define oscillator strength (f_{ki}). [1]

Give the oscillator balance equation for transitions between the levels $|i\rangle$ and $|k\rangle$ allowing for the degeneracy of the levels. [2]

The transition probability A_{ki} for spontaneous emissions from $|k\rangle \rightarrow |i\rangle$ is

$$A_{ki} = \left[\frac{e^2 2}{3\varepsilon_0}\frac{\omega_{ki}^3}{hc^3}\right]\left|\langle i|r|k\rangle\right|^2.$$

Derive an expression for A_{ki} involving the statistical weights of the levels. Explain carefully the influence of the degeneracy of the upper, $|k\rangle$, and lower, $|i\rangle$, states. [4]

Determine A_{ki} for a $^1P_1 \rightarrow$ $^1S_0$ transition in the optical region at 600 nm where the total matrix element for the transition is $4a_o$. [4]

Derive an expression for f_{ki} in terms of the degenerate transition matrix elements.
Assume the radiative decay rate to be $\dfrac{e^2\omega^2}{6\pi\varepsilon_0 m_e c^3}$. [3]

Atomic hydrogen is in the 3d state. Which transitions to the *np* and *mf* states are allowed and which have negative oscillator strengths? [3]

Given Σ_n (3d $\rightarrow$ *np*) = $-$ 0.402 and Σ_m (3d $\rightarrow$ *mf*) = 1.302
use the Thomas-Kuhn-Reiche sum rule to determine the fraction of the oscillator strength in the continuum. [3]

2. What conditions must apply during the atomic excitation for Quantum Beats to be observed? [2]

Consider an atom of ground state $|g\rangle$ with the excited states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ undergoing coherent excitation. Derive an expression for the observed intensity of the Quantum Beats in terms of the decay constants and frequency separation of the states. [10]

Describe a beam foil apparatus which can be used to measure quantum beats. [4]

A helium ion, He^+, is incident on a foil of thickness 100 nm.
Charge capture in the foil produces helium 3 $^3P_{0,1,2}$ in which the fine structure separation of the J = 1 and J = 2 levels is 658 MHz.

What is the minimum kinetic energy of the He^+ beam required in order to

(i) produce coherent excitation of the J = 1 and J = 2 levels and

(ii) observe the quantum beats if the resolution of the apparatus is 10^{-3} m? [4]

3. Explain what is meant by a Virtual state. [2]

Derive expressions for the two photon ionization rate of an atom as a function of laser intensity I when the intermediate state is (a) Real and (b) Virtual. [4]

For threshold two photon ionization estimate the ratio of the transition probabilities when the intermediate state is real compared to when it is virtual. Assume an ionization potential of 3 eV. [2]

If a real state exists at 1 eV, determine the life time of the virtual state in threshold two photon ionization. [2]

Explain how the Doppler width is reduced to first order in two photon absorption spectroscopy. [4]

Describe an experiment to measure the hyperfine separations in the $3^2S_{1/2} \rightarrow 5^2S_{1/2}$ two photon transition in sodium. The nuclear spin of sodium is 3/2. [6]

4. Distinguish between Longitudinal and Transverse modes in a laser cavity. [2]

Show that the mode spacing, $\Delta\omega$, of Longitudinal modes in a laser cavity of length L is $\Delta\omega = \pi c L^{-1}$. [2]

If the coherence length of the radiation is λ_c, what is the mode distribution when $\lambda_c >> L$ and when $\lambda_c << L$? Give expressions for the average intensity in each case when the distribution of the light is Lorentzian. [4]

Show that the fringe visibility, V, in a Young's experiment can be written as

$$V = \frac{2u_1 u_2^* \exp[-\gamma \Delta s / c]}{|u_1|^2 + |u_2|^2},$$

where γ is the band width of the radiation, Δs is the path length difference and u_1 and u_2 are geometric factors.

Assume the first order correlation function to be

$$2I(\varepsilon_0 c)^{-1} \exp(i\omega\tau - \gamma|\tau|),$$

where ω is the radiation frequency, I is the light intensity and τ is the path length time difference. [6]

How is V influenced by the use of coherent or chaotic light? [2]

When $\omega = 3\times10^{15}$ Hz and $\gamma = 6\times10^{11}$ Hz how many intensity oscillations will occur at $\exp(-2)$ of the maximum visibility? [4]

5. Explain how optical molasses cooling works and the role of frequency chirping. [3]

Sodium, ^{23}Na, is being cooled in a molasses experiment. The lifetime of the sodium transition is 16×10^{-9} s and the frequency is 5×10^{14} Hz.

(i) Determine the velocity change of a sodium atom associated with a single photon recoil. [1]

(ii) Estimate the number of absorption and emission cycles required to cool the sodium from 500 K to 0.6 K. [1]

(iii) How long will it take to cool the sodium to 0.6 K? [1]

Explain what is meant by the Doppler cooling limit. [4]

Consider sodium in two counter-propagating resonant laser beams which have a damping coefficient β. When the detuning δ is negative, show that the steady state kinetic energy is

$$\frac{h\gamma}{16\pi}\left[\frac{2|\delta|}{\gamma}+\frac{\gamma}{2|\delta|}\right],$$

and determine an expression for the Doppler cooling limit temperature. [10]

Take $$\beta=\frac{8hk^2\delta}{\pi\gamma\left[1+\left(\frac{2\delta}{\gamma}\right)^2\right]},$$

where γ is the line width and k is the wave vector.

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