UNIVERSITY COLLEGE LONDON

EXAMINATION FOR INTERNAL STUDENTS

MODULE CODE : **PHAS3774**

MODULE NAME : **Topics in Modern Cosmology**

DATE : **10-May-07**

TIME : **14:30**

TIME ALLOWED : **2 Hours 30 Minutes**

2006/07-PHAS3774A-001-EXAM-12

Answer ALL questions from SECTION A and TWO questions from SECTION B

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

1 Mpc $= 3.086 \times 10^{19}\,\mathrm{km}$

1 yr $= 3.156 \times 10^{7}\,\mathrm{s}$

Speed of light $c = 3 \times 10^{5}$ km s^{-1}

Constant of gravitation $G = 6.672 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Radiation constant $\alpha = 7.565 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$

SECTION A

1. Outline the observational evidence for the isotropy and homogeneity of the Universe. [6]

2. Show that the redshift z of a distant galaxy is related to the scale factor $a(t)$ at that epoch by $1 + z = 1/a$, assuming that at the present epoch $a(t_0) = 1$. [7]

3. Assume that matter and radiation evolve independently, and that their densities evolve like $\rho_m \propto a^{-3}$ and $\rho_r \propto a^{-4}$, respectively. Express the redshift at which they contribute equally to the total energy density in terms of the ratio of the matter and radiation densities at the present epoch. Assuming that the present-epoch radiation density is known from observations, show that the above ratio scales like $\Omega_m H_0^2$, where Ω_m is the matter density parameter and H_0 is the Hubble constant, both at the present epoch. [6]

4. Describe *three* independent pieces of evidence that support the Hot Big Bang model. [7]

5. Plot and label a typical light curve of a Supernova Type Ia and explain how it can be used to estimate its luminosity distance.

   Explain how the the Supernovae Ia data can be used to test the existence of Dark Energy. [7]

6. Explain briefly the evidence for dark matter in clusters of galaxies based on the velocity dispersion of cluster galaxies. [7]

SECTION B

7. (a) The Friedmann equation describing the expansion of the Universe is given by (using standard notation)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}$$

and the acceleration equation is given by

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3}.$$

Using these two equations, derive the fluid equation describing the evolution of the density ρ of material in the Universe:

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$$

[10]

(b) Assuming that the Universe is flat, matter-dominated ($p = 0$) and $\Lambda = 0$, solve the Friedmann and fluid equations to obtain: **[12]**

$$a(t) = \left(\frac{t}{t_0}\right)^{2/3}; \quad \rho(t) = \rho_0\left(\frac{t_0}{t}\right)^2; \quad \text{and} \quad t_0 = \frac{2}{3H_0}.$$

(c) Consider the acceleration equation for a Universe with matter ($p = 0$) and $\Lambda > 0$. Show that the acceleration would only take place for redshift $z < z_{acc}$, where

$$z_{acc} = (2\Omega_\Lambda/\Omega_m)^{1/3} - 1.$$

Evaluate z_{acc} for a flat Universe with $\Omega_\Lambda = 0.75$. **[8]**

8. (a) Consider the the Friedman equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda}{3}.$$

for the epoch when Universe was radiation-dominated, i.e. $a \propto t^{1/2}$ and $\rho = \alpha T^4/c^2$, where α is the radiation constant.

State clearly why the curvature and Λ terms can be neglected in the early Universe. [5]

Show that then the age of the Universe can be related to the photon temperature, T, via [10]

$$t = \left(\frac{3c^2}{32\pi\alpha G}\right)^{1/2} \frac{1}{T^2} = \frac{2.3 \times 10^{20}}{T^2} \text{ sec.}$$

How old was the Universe when the temperature was 10^9 K? [2]

(b) The entropy for relativistic gas ($p = \frac{1}{3}\rho c^2$) can be written as

$$S = \frac{4}{3}\frac{\rho c^2}{T}$$

and the energy density of a relativistic gas composed of electrons, positrons and photons (e^+, e^-, γ) is given by

$$\rho c^2 = \frac{7}{4}aT_e^4 + aT_\gamma^4,$$

where T_e, T_γ are the electron and photon temperature, respectively.

Assuming thermal equilibrium, calculate the adiabatic change of T_γ immediately after e^+/e^- annihilation. [13]

9. (a) In linear theory the evolution of the mass density contrast δ is given by:

$$\ddot{\delta} + 2\dot{\delta}\frac{\dot{a}}{a} - 4\pi G\rho_m\delta = 0 \qquad \text{(equation 1)}$$

where $a(t)$ is the scale factor, and ρ_m is the mean matter density.

Discuss briefly the role of the second term. [3]

Derive the growing solution in two cases:

(i) for the case of a static universe, [5]

and

(ii) for an Einstein-de Sitter universe ($\Omega_m = 1, \Omega_\Lambda = 0$), in which $a \propto t^{2/3}$. [5]

Comment on why the results for cases (i) and (ii) are different. [2]

(b) Suppose the universe contains an unstable elementary particle that decays after the recombination epoch. It would produce relativistic remnants and the universe would re-enter a radiation dominated phase characterised by radiation density $\rho_r \propto a^{-4}$. Using the relation $H^2 = (8\pi G/3)\rho_r$ show that after the decay era $a \propto t^{1/2}$ and the Hubble parameter obeys [5]

$$H(t) = \frac{1}{2t}.$$

Give the general solution of equation (1) in part (a) for this case, assuming that the last term on the left hand side of equation (1) is negligible. [10]

10. (a) Describe the main properties of the Cosmic Background Radiation (CBR). [10]

(b) Explain how can we test if the Universe is flat from the angular power spectrum of the CBR. How does the concept of Inflation explain the flatness of the Universe? [7]

(c) (i) If the current age of the Universe is 14 billion yrs, estimate how far light has travelled since the Big Bang. Give your answer in Mpc. [3]

(ii) Assume that the Universe has always been matter-dominated, and the CBR has been travelling towards us uninterrupted since decoupling. Estimate the age of the Universe at decoupling, when $a = 10^{-3}$, and the distance that light could have travelled up to the time of decoupling. [3]

(iii) What is the physical size of this region today in Mpc? [2]

(iv) Hence, assuming that the distance to the origin of the CBR is given by part (i) above, what is the angle subtended by the distance light could have travelled before decoupling? [3]

(v) How would this angle change if decoupling took place at $a = 10^{-2}$? [2]