UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sc.* *M.Sci.*

Physics 3C43: Lasers and Modern Optics

COURSE CODE : **PHYS3C43**

UNIT VALUE : **0.50**

DATE : **02-MAY-06**

TIME : **10.00**

TIME ALLOWED : **2 Hours 30 Minutes**

06-C1176-3-30

TURN OVER

Answer all 6 questions from section A and 2 questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of marks for each question or subsection of a question.

FORMULAE AND CONSTANTS

$c = 3 \times 10^8$ m/s ; $k = 1.38 \times 10^{-23}$ J/K

$h = 6.64 \times 10^{-34}$ J·s

SECTION A

1. Derive the ray-transfer matrix for a thin lens in terms of its focal distance f with the aid of an appropriate diagram. [6]

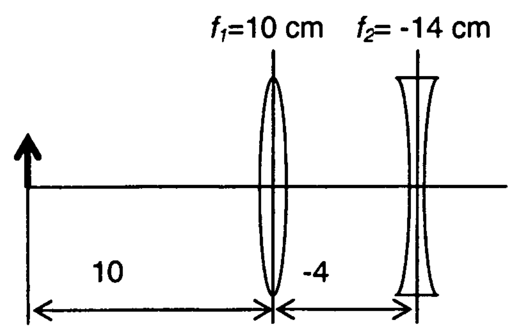
2. What is meant by the Doppler linewidth? [2]
   What is the temperature dependence of the Doppler broadening? [2]
   What is the natural linewidth of a spectral line? Give a symbolic, quantitative expression of the energy linewidth. [4]

3. What is the fundamental hypothesis of "geometrical optics"?
   What major physical phenomenon is possible to disregard as a consequence of such hypothesis?
   In the context of ray-transfer matrix optics, what is the sign convention for the measurement of angles? [6]

4. Explain the difference between dichroic and birefringent materials. Give a definition of "optically isotropic" material. Can a birefringent material be optically isotropic? [5]

5. With reference to a multilevel atomic or molecular system (i.e. a system with a number of energy levels greater than 1) define "population inversion". [2]
   Explain why it is easier to produce inversion population, and therefore lasing, in a so-called "4-level" system compared to a "3-level" system. [3]
   Calculate the ratio of the populations at equilibrium at 300 K for two non-degenerate levels with an energy difference of 3 eV (assume the degeneracies to be 1). [3]

6. Consider the optical system sketched in the figure below, where the thick arrow indicates the object position.



Find the position of the image. [7]

SECTION B

7. Two converging thin lenses L_1 and L_2 with the same focal length f and with their centres located along the same optical axis (i.e. they are concentric) are separated by a distance $d = f/2$.

   a) Draw a schematic diagram of the lenses. [1]

   b) By using only the thin lens equation (no matrices) locate the image position for an object placed at distance 4f to the left of L_1 and draw the position on the schematic diagram. [6]

   c) Find the focal points of this lens combination treated as a single thick lens. [7]

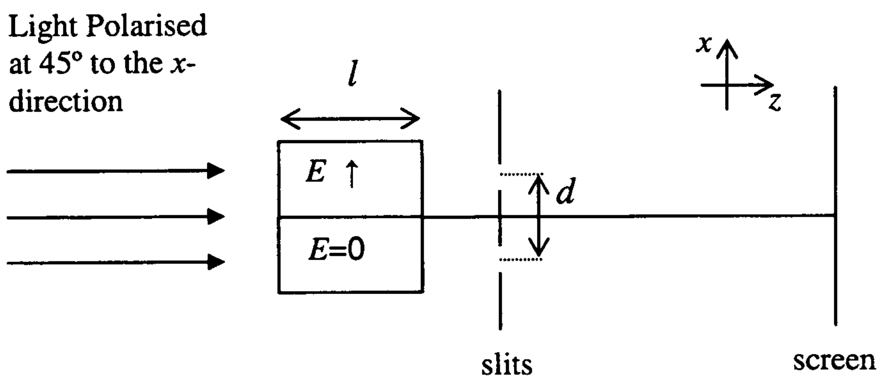
   d) Find the ray-transfer matrix of the system in terms of the focal length f. [8]

   e) Verify that the expressions: $f_1 = D/C$ *and* $f_2 = -A/C$ of the foci distance from the input/output plane in terms of the ray-transfer matrix coefficients (with the usual convention for the naming of the ray-transfer matrix coefficients, i.e. $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$), give the same results as in c) above. [4]

   f) With the aid of a simple diagram explain why the magnification on the principal planes is m=1. Use this property to find the position of the principal planes. [4]

8. A beam of light with wavelength λ = 600 nm, travelling in the z-direction, is polarised at 45° to the x-direction. The beam then passes through a Kerr cell whose main axes are oriented along x and y. The refractive index in these directions is n_x and n_y, respectively. The cell is 1 cm long and the Kerr constant at 600 nm is $K = 4.4\times10^{-12}$ m V^{-2}.

a) What is the dependence between n_x, n_y K and the electric field E? [3]
b) Explain how the Kerr cell changes the polarisation state of the light. [2]
c) If the light, after passing through the Kerr cell, goes through a polarisation analyser whose plane of polarisation is perpendicular to that of the original beam, calculate the smallest value of E that gives maximum transmission. [13]
d) What is the state of polarisation of the light emerging from the Kerr cell if the value of E^2 is half that calculated in a)? [3]
e) Consider the arrangement described in the figure below:



The electric field is applied to the upper half of the Kerr cell only, and, after passing through the Kerr cell, the light goes through two slits as shown (there is no polarisation analyser after the slits). Assuming that the electric field affects n_x but not n_y, calculate the values of E^2 such that the phase angle difference (δ) between the x and y components of the beam through the upper slit is $\delta = 2m\pi$, with m an integer. Then discuss the contrast or modulation of the interference pattern, at a large distance beyond the slits, in this case ($\delta = 2m\pi$) and for the case when $\delta = (2m+1)\pi$.
Remember that the modulation or contrast is defined as:

$$modulation = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

where I_{max} and I_{min} are the maximum and the minimum intensities of the light in the interference pattern. [9]

9. Consider a planar Fabry-Perot (FP) cavity with mirrors spaced a distance L, which are characterised by reflection and transmission coefficients r and t, respectively, and for which the following relations hold true:

$$T = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(\delta/2)}$$

$$\text{where} \quad \delta = 2\left(\frac{2\pi}{\lambda}\right)n' L \cos\theta'$$

a) Specify the meaning of all the undefined symbols in the expression above, and, where appropriate, express them as a function of r and t. What is the name of the function describing T? [5]

b) Draw a diagram of the FP cavity and a generic ray incident on the cavity, indicating relevant quantities such as angles and/or lengths which appear in the equations above. Sketch also the secondary rays generated by partial reflection at the various interfaces and their subsequent propagation/reflection/refraction at the mirrors' surfaces (e.g. up to 5 internal reflections). [5]

c) For $\theta = 60°$, $n' = 3n$ and $R = 1 - \sigma$ (with $\sigma \ll 1$), calculate the frequency of the maxima of the T function above and the free spectral range $\Delta\nu_{FSR}$ of the cavity as a function of n and L. Give the definition of the cavity finesse in terms of the full width at half maximum $\Delta\nu_{FWHM}$, and of the free spectral range $\Delta\nu_{FSR}$, and find an expression of the finesse in terms of R. [16]

d) Calculate the reflectivity of the mirrors of a planar FP cavity needed for achieving a finesse of 10^3. [4]

10. Consider a flash-lamp pumped, Q-switched laser.

a) Explain the basic principle of the generation of the short pulses with the aid of a schematic diagram of the following quantities as a function of time: (i) the flash lamp output; (ii) the cavity Q; (iii) the population inversion; (iv) the output. [10]

b) List at least two methods with which Q-switching can be accomplished [2]

c) Estimate the energy of the output pulses from a Q-switched laser as a function of the lasing transition frequency ν, the volume of the laser medium V, the population inversion before the cavity is switched on (N_i), and of the population inversion at the end of the pulse (N_f). Calculate the numerical value of the output energy for $N_i = 10^{24}$ m^{-3}, $N_i >> N_f$, $V = 10^{-5}$ m^3, and ν = 500 THz. [4]

d) Derive a symbolic expression of the pulse duration t_c as a function of the parameters introduced part c) above, and of the mirrors' reflectivity R. Assume that the refractive index n of the laser medium is 1, and the cavity length is L. [6]

e) Calculate the numerical value of the pulse duration for a cavity length L =0.3 m and a mirror reflectance of 0.9. [2]

f) Calculate the average power in the pulse. [2]

g) Calculate the cavity length L so that the average power output *0.85 GW*. [1]

h) Suppose the length calculated in part g) above is not suitable because of practical reasons. Provide an alternative design of the laser resonator to achieve the same increase in average power output while keeping L=0.3 m. [3]