**UNIVERSITY COLLEGE LONDON**

University of London

**EXAMINATION FOR INTERNAL STUDENTS**

For The Following Qualifications:-

*B.Sc.* *M.Sci.*

**Physics 3C43: Lasers and Modern Optics**

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| COURSE CODE | : | **PHYS3C43** |
| UNIT VALUE | : | **0.50** |
| DATE | : | **04-MAY-05** |
| TIME | : | **10.00** |
| TIME ALLOWED | : | **2 Hours 30 Minutes** |

05-C1094-3-70


**TURN OVER**

**Answer all 6 questions from section A and 2 questions from section B.**

**The numbers in square brackets in the right-hand margin indicate the provisional allocation of marks for each question or subsection of a question.**

FORMULAE AND CONSTANTS

$c = 3 \times 10^8$ m/s
$k = 1.38 \times 10^{-23}$ J/K
$h = 6.64 \times 10^{-34}$ J·s

## SECTION A

1. In the context of ray transfer matrices, state the paraxial approximation. [2]
   State any other approximation that is usually made in the context of ray-transfer matrix optics. [2]
   Given that ray transfer matrices are two-dimensional, is this dimensionality a consequence of the paraxial approximation? [2]

2. Define the first and the second focal points for a complex optical system, such as a thick lens. [3]
   A beam is propagating from left to right and encounters two optical subsystems (1, and then 2) characterised by the matrices $M_1$ and $M_2$ respectively. Give the optical matrix of the system constituted by the two subsystems. [2]
   Give the condition on the optical matrix for the whole system to behave as a telescope. [3]

3. With the aid of a schematic energy level diagram give a brief explanation of the working mechanism of the Ar-ion laser. [3]
   Explain the different mechanisms for populating the laser level. [3]

4. An Ar-ion laser whose emission has been tuned on the 457.9 nm line has an internal beam waist of 0.4 mm. Determine the beam divergence. [3]
   By what factor would the divergence improve if we tuned the laser on the 363.8 nm line? [2]
   Calculate the brightness of a 1 mW-beam at the wavelength of 363.8 nm. [3]

5. Define the Faraday effect. [2]
   Give a quantitative expression of the rotation of the polarisation angle due to the Faraday effect in terms of the Verdet constant $V$ and of the other relevant physical quantities. [2]
   Consider a beam at 589 nm entering a cube of ZnS ($V$=0.225 min/Gcm) whose side is 10 cm long, and that is subject to a field of 9 G. What is the rotation angle at the exit of the cube? [2]

6. The non-linearity of the refractive index n (for crystalline materials) is usually expressed as

$$\frac{1}{n^2} = \frac{1}{{n_0}^2} + rE + RE^2$$

where $r$ and $R$ are constant for the medium.

What does the symbol $E$ stand for? [1]

Is the Kerr effect related to the linear or non-linear term (in $E$) in the equation above? [2]

Give the expression that describes the difference between extraordinary and ordinary refractive index in terms of the intensity and wavelength of the radiation. [3]

## SECTION B

7.

a) State the meaning of the symbols in the ray transfer matrix of a refracting spherical surface expressed as

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'}-1\right) & \frac{n}{n'} \end{bmatrix}$$

[2]

b) Find the numerical value of the ray transfer matrix for a thick convex lens, in air, and for which the radius of curvature is 10 cm, the refractive index is 4, and the thickness $t$ is 0.3 cm. [8]

c) With reference to a generic complex optical system, derive a symbolic expression for the distance of the first focus from the first principal plane in terms of the generic ray transfer matrix expressed as follows:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

[4]

d) Consider an optical system consisting of two thin lenses $A$ and $B$ made of the same glass with refractive index $n = 1.6$ and with radii $R_{A1} = -R_{A2} = 6$ cm and $R_{B1} = 8$ cm, $R_{B2} = -12$ cm.

- Calculate the ray transfer matrices $M_A$ and $M_B$ for both lenses. [6]
- Determine an expression for the effective focal length of the system in terms of $L$ and the lenses' focal lengths $f_A$ and $f_B$, when the lenses are spaced a distance $L$. Write down the numerical value of the equivalent focal length if $L$ is 5 cm. [4]
- Find a symbolic expression in terms of the radii of curvature of the lenses, the distance $L$, and the refractive index $n$, for which $\frac{d(1/f_{eq})}{dn} = 0$, where $f_{eq}$ is the equivalent focal length (assume the lenses are in air). What is the numerical value of $L$ in the case of the system discussed in this question? [6]

8.

a) A laser cavity is constituted by a planar Fabry-Perot etalon whose mirrors have reflectivity $R_1 = 0.91$ and $R_2 = R_1/2$ respectively. The separation of the mirrors is $L = 1$m. The laser medium is characterised by a loss coefficient $\gamma = 10\ \text{m}^{-1}$. The lasing transition occurs at a wavelength of 8 μm, and it is inhomogeneously broadened with a linewidth $\Delta\nu = 50$MHz. The Einstein $A$ coefficient of the upper laser level is 150 $\text{s}^{-1}$. Derive a symbolic expression for the round trip gain of the cavity in terms of the reflectivities of the resonator's mirrors, the cavity length, and the gain and loss coefficient. [5]

b) Derive an expression for the threshold small signal gain coefficient $k_{th}$. [4]

c) Calculate the numerical value of $k_{th}$ for the values of $R_1$, $R_2$ and $\gamma$ given above. [2]

d) Derive the steady state relation between the value of the small signal gain coefficient at threshold and at saturation. What would happen if this relation was not satisfied? [2]

e) State the meaning of the various symbols in the expression of the gain coefficient given below and calculate the value of the inversion population (in $\text{cm}^{-3}$) necessary to reach threshold.

$$\kappa(\nu) = g(\nu)\left(N_2 - \frac{g_2}{g_1}N_1\right)B_{21}\frac{nh\nu}{c}$$ [8]

f) Consider a two-level system for which only absorption and spontaneous emission are possible. Assume also that the ratio of the populations of the two levels follows a Boltzmann statistics with degeneracies equal to 1 for both levels. Show that the density of radiative energy ($\rho(\nu)$) per unit frequency $\nu$ that can be calculated for such a system, is inconsistent with Planck's equation:

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$ [9]

Where $h$ is Planck's constant, $c$ is the speed of light, $k$ is Boltzmann's constant, and $T$ the absolute temperature.

9. An Ar-ion laser beam at 514 nm is focused by a thin lens of focal length $f = 10$ cm. The lens is located at a distance $z = 30$ cm from the original beam waist, which is located at $z = 0$. The beam size at $z = 10$ cm is 500 μm.

a) Derive an expression for the size of the original beam waist, and calculate its size. [10]

b) Using the *ABCD* system matrix determine the position of the new beam waist. [10]

c) Calculate the size of the new beam waist. [4]

d) Calculate what happens to the size and position of the beam waist if the focal length of the thin lens is doubled. [4]

e) Give the value of the collimated beam length. [2]

10. Consider a thin convex lens with focal length $f$.

a) Derive an expression for the magnification of an object that is placed at a distance $d < f$ from the lens. [5]

b) Derive an expression for the total magnification when the same object (at the same distance $d$) is looked at through two coaxial and adjacent thin lenses, of focal lengths $f_1$ and $f_2$. [2]

c) In the case $f = 5$ cm, $f_1 = 5$ cm, $f_2 = 4$ cm, and $d = 3$ cm find the numerical value of the magnification in the cases discussed in a) and in b) above. [6]

d) With the aid of a labelled diagram explain and comment the nature of the image in the cases $d < f$ and $d > f$. [6]

e) An image of an object is formed on a screen by a lens. Leaving the lens position fixed, the object is moved to a new position and the image screen is moved until it again receives a focused image. If the two object positions are $d_1$ and $d_2$ and if the transverse magnification of the image is $M_1$ and $M_2$ respectively, show that the focal length of the lens can be written as:

$$f = \frac{d_2 - d_1}{\frac{1}{M_1} - \frac{1}{M_2}}$$ [8]

f) Under which condition is $M_1 = M_2$? [2]
Why is the equation derived in 10.e) no longer usable if $M_1 = M_2$? [1]