UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 3C25: Solid State Physics

COURSE CODE	:	**PHYS3C25**
UNIT VALUE	:	**0.50**
DATE	:	**07-MAY-04**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

04-C1093-3-90

TURN OVER

Answer EVERY question from section A and TWO questions from section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Mass of the electron	m_e	=	9.11×10^{-31} kg
Charge on the electron	e	=	-1.602×10^{-19} C
Planck's constant/2π	$\hbar$	=	1.05×10^{-34} J s

SECTION A

[Part marks]

1. The energy per ion pair in an ionic crystal may be written as

$$E_b = \frac{A}{r^{12}} - \frac{\alpha_M Z^2 e^2}{4\pi\epsilon_0 r},$$

where r is the nearest-neighbour separation. Explain the physical significance of the two terms in the energy, and state the meaning of Z. **[2]**

Obtain an expression for the equilibrium value of the nearest-neighbour separation, and hence show that binding energy per ion pair in equilibrium is $11A\left(\alpha_M Z^2 e^2/(48\pi\epsilon_0 A)\right)^{12/11}$. **[4]**

2. Write down the equation expressing Bragg's law relating the angle θ at which radiation of wavelength λ is reflected from crystal planes a distance d apart, explaining the significance of any other symbols you use. **[3]**

Draw diagrams showing the arrangement of lattice points in a simple cubic crystal, a body-centred cubic crystal, and a face-centred cubic crystal, looking down the z-axis. Draw points in the $z = 0$ plane as solid dots, points half-way up the cell as open circles. Use your diagrams to explain why a first-order (100) reflection would not be expected from the body-centred or face-centred cubic crystals. **[4]**

3. Sketch the dispersion relation for longitudinal phonons on a linear monatomic chain. In a separate diagram sketch the dispersion relation for a chain made up of alternating atoms with two different masses. **[2]**

Write down the conservation equations which apply to the interaction of two phonons to form one resultant phonon. Explain what is meant by an *umklapp process*. What is the significance of umklapp processes in thermal conduction? **[4]**

4. If the relaxation times describing the effects of impurity scattering and phonon scattering on the electrical conductivity of a metal are τ_i and τ_p respectively, and the scattering processes may be assumed to be independent, write down an expression for the resultant scattering time τ when both processes act. **[2]**

What dependence on temperature would you expect for τ_i and for τ_p? The electrical resistivity of platinum varies with temperature as shown in the table below: discuss whether this is consistent with your expected temperature dependence of τ. **[5]**

Temperature (K)	273.2	373.2	573.2	973.2	1473.2
Resistivity (10^{-8} ohm m)	9.81	13.60	21.00	34.30	48.30

5. Write down an expression for the kinetic energy E_k of a free electron in terms of its wavevector k, and show how the electron mass is related to a derivative of E_k. What is meant by the effective mass of a carrier in a semiconductor, and why is it different from the mass of a free electron? **[4]**

The energy of an electron in the valence band of a certain semiconductor may be written as

$$E = E_\mathrm{v} - W[1 - \cos(ka)],$$

where E_v, W and a are constants. Sketch the variations of the energy and of the effective mass of the electron as functions of k. **[3]**

6. Long cylindrical specimens of Type I and Type II superconductors are aligned parallel to a magnetic field with flux density $\mathcal{B}$. Sketch the dependence on $\mathcal{B}$ of the average magnetisation $\mathcal{M}$ inside each specimen. **[3]**

Explain the term *superconducting energy gap*, and give brief descriptions of two phenomena which provide evidence for the existence of the energy gap. **[4]**

SECTION B

7. State the assumptions of the Debye model for the specific heat arising from phonons in a solid. Hence derive an expression for the thermal energy of a three dimensional insulating solid. You may use the fact that for a solid of volume V the number of allowed values of the wavevector **k** for which the modulus of the wave vector, k, lies between k and $k + \mathrm{d}k$ is $V/(2\pi)^3\, 4\pi k^2 \mathrm{d}k$. **[18]**

   Given that
$$\int_0^\infty \frac{x^3}{e^x - 1}\mathrm{d}x = \frac{\pi^4}{15},$$
   show that the specific heat at low temperatures is proportional to the cube of the absolute temperature. **[6]**

   Without detailed calculation, explain what temperature dependence would be expected for the specific heat of a one-dimensional chain and of a planar solid. **[6]**

8. Give two examples of electronic properties of materials that can be explained by the free electron model, and two which cannot. **[6]**

   Derive an expression for the Fermi energy of a metal in terms of the free electron concentration. You may use the fact that for a solid of volume V the number of allowed values of the wavevector **k** for which the modulus of the wave vector, k, lies between k and $k + \mathrm{d}k$ is $V/(2\pi)^3\, 4\pi k^2 \mathrm{d}k$. **[12]**

   Sodium is monovalent, and forms body-centred cubic crystals in which the sides of the cubic unit cell are 0.425 nm long. Calculate the Fermi energy of sodium, assuming the free electron model. **[6]**

   What is the reciprocal lattice of the body-centred cubic lattice? The free electron Fermi surface for the monovalent body-centred cubic metal lies entirely within the first Brillouin zone. Along which directions in reciprocal space is the Fermi surface closest to the Brillouin zone boundary? **[6]**

9. Explain with the aid of diagrams what is meant by the terms *direct* and *indirect* interband transitions in a semiconductor. **[6]**

The density of states (including the spin degeneracy factor) for electrons with energy E in the conduction band of a semiconductor of unit volume is

$$g(E) = \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{\frac{3}{2}} (E - E_c)^{\frac{1}{2}},$$

where E_c is the energy of the conduction band minimum and m_e^* is the electron effective mass in the conduction band. Write down the corresponding expression for holes in the valence band. Stating clearly any assumptions, derive the law of mass action

$$n_e n_h = f(T) e^{-E_g/k_B T},$$

which states that the product of the electron and hole concentrations (n_e and n_h respectively) in a given semiconductor is a characteristic function of the temperature and the band gap E_g. Give an expression for the function $f(T)$. Note that **[16]**

$$\int_0^\infty x^{1/2} e^{-x}\, dx = \left(\frac{\pi}{4}\right)^{1/2}.$$

Hence derive an expression for the variation of the chemical potential, μ, with temperature in an undoped sample of the semiconductor. **[8]**

10. Explain what is meant by the *domain structure* of a ferromagnetic material. What factors influence the thickness of a domain wall? **[6]**

A solid contains n ions per volume, each having a single unpaired electron which may be treated as having a spin of $\frac{1}{2}$ with gyromagnetic ratio $g = 2$ and no orbital angular momentum. Show that in a magnetic flux density $\mathcal{B}$ at temperature T the material will acquire a magnetisation

$$\mathcal{M} = n\mu_B \tanh\left(\frac{\mu_B \mathcal{B}}{k_B T}\right).$$

Show that this is consistent with Curie's law at high temperature or low field. **[9]**

Write down an expression for the energy associated with the interaction of this average magnetic moment with the field. Hence deduce the magnetic contribution to the specific heat of the material. **[6]**

If the field $\mathcal{B}$ arises from exchange interactions between spins and has the form $\lambda\mathcal{M}$, show that the material will acquire a spontaneous magnetic moment below a critical temperature T_c, and derive an expression for that temperature. **[9]**

END OF PAPER