UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 2B29: Electromagnetic Theory

COURSE CODE : **PHYS2B29**

UNIT VALUE : **0.50**

DATE : **16-MAY-06**

TIME : **14.30**

TIME ALLOWED : **2 Hours 30 Minutes**

06-C1169-3-30

TURN OVER

Answer ALL SIX questions in Section A and THREE questions from Section B

The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may find the following constants and theorems useful.

$$\begin{aligned}
\epsilon_0 &= 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}} \\
\mu_0 &= 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}} \\
\text{Speed of light } \textit{in vacuo} &= 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}} \\
\text{Electron mass} &= 9.11 \times 10^{-31}\,\mathrm{kg} \\
\text{Electron charge} &= 1.60 \times 10^{-19}\,\mathrm{C}
\end{aligned}$$

For any vector field $\mathbf{F}$ and scalar field φ :

$$\begin{aligned}
\nabla \cdot \nabla \times \mathbf{F} &= 0 \\
\nabla \times \nabla\varphi &= 0 \\
\nabla \times (\nabla \times \mathbf{F}) &= \nabla(\nabla \cdot \mathbf{F}) - \nabla^2\mathbf{F} \\
\oint_C \mathbf{F} \cdot d\boldsymbol{\ell} &= \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{s} \\
\oint_S \mathbf{F} \cdot d\mathbf{s} &= \int_V (\nabla \cdot \mathbf{F})\, dv
\end{aligned}$$

SECTION A

1. Write down the expression for the e.m.f. induced in a closed conducting circuit C by the rate of change of the magnetic flux Φ_C which passes through it. Explain how this is related to the integral form of the Faraday law. [3]

   Use the Stokes theorem to derive the differential form of the Faraday law

   $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

   from the integral form, giving a brief justification for each step. [3]

2. In a linear magnetic medium, we can write:

   $$\begin{aligned}
   \mathbf{J}_M &= \nabla \times \mathbf{M} \\
   \mathbf{j}_M &= \mathbf{M} \times \mathbf{n}
   \end{aligned}$$

   Define the terms $\mathbf{J}_M$, $\mathbf{j}_M$ and $\mathbf{M}$ and explain qualitatively (at the atomic level) how $\mathbf{J}_M$ and $\mathbf{j}_M$ arise (you may find a sketch helpful). [7]

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3. The wave equation for a plane wave is:

$$\nabla^2\mathbf{E} - \sigma\mu\frac{\partial\mathbf{E}}{\partial t} - \epsilon\mu\frac{\partial^2\mathbf{E}}{\partial t^2} = 0$$

Define the term: σ [2]

Given a plane wave of the form

$$\mathbf{E} = \mathbf{E}_0 \exp i\,(\mathbf{k}\cdot\mathbf{r} - \omega t)$$

then

$$\nabla^2\mathbf{E} = -k^2\mathbf{E}.$$

Calculate $\partial\mathbf{E}/\partial t$ and $\partial^2\mathbf{E}/\partial t^2$. [2]

Show that the dispersion relation is:

$$k^2 = \mu\epsilon\omega^2\left(1 + \frac{i\sigma}{\epsilon\omega}\right)$$

and explain what happens if $\sigma >> \epsilon\omega$. [3]

4. Show, starting with the appropriate Maxwell equation, that normal components of **B** are continuous across a plane boundary between media with different magnetic properties. [6]

5. We can write the Maxwell equation $\nabla\cdot\mathbf{E} = \rho/\epsilon_0$ as $\nabla\cdot\mathbf{D} = \rho_f$.

Define ρ and ρ_f and explain the difference between them. [4]

How is the polarisation defined in terms of the polarisation charge density (both bulk and surface)? Give appropriate equations, defining the terms where necessary. [3]

6. Explain *qualitatively* the difference between a hard and a soft ferromagnetic material. [2]

Sketch hysteresis loops (B vs H) for soft and hard materials, indicating the points used to define coercivity and remanence. [5]

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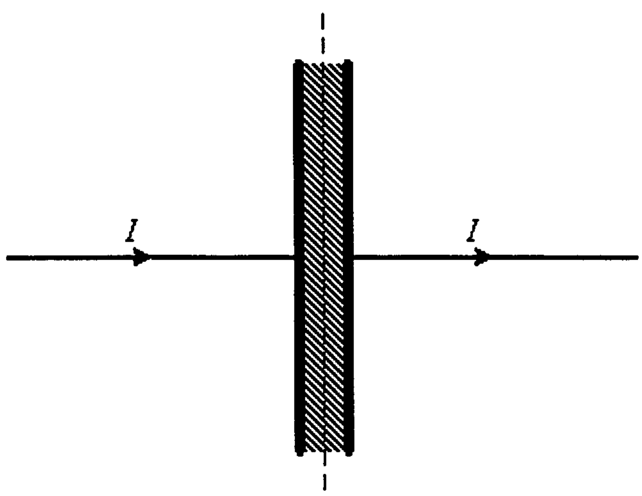
SECTION B

7. Use appropriate defining equations to derive the dimensions of the following electromagnetic quantifies in terms of the basic dimensions of mass [M], length [L], time [T] and charge [Q]:
   Electric field strength **E**, potential difference V, capacitance C,
   electric polarization **P**, electric induction **D**, permittivity of a medium ε. [6]

   Define the *dielectric constant* κ in terms of the behaviour of a parallel plate capacitor when the gap between the plates is initially empty and is then filled with dielectric material. How is the dielectric constant related to the permittivity of the medium between the plates? Use an integral form of the Gauss law to show how the magnitude of **D** inside the capacitor depends on the charge on the plates and their area. [6]

   Give an expression for the displacement current density in terms of the value of **D** at a point. What is its value inside a capacitor with circular plates of radius 4 mm and an empty gap of width $d = 0.02$ mm when a current of 0.5 A is flowing in through straight wires connected to the centres of the plates, as shown? [3]



   Derive a formula for the magnitude B of the magnetic induction between the plates of this circular capacitor in the plane P at a distance of r from the centre. You may assume that the relative permeability $\mu_r = 1$. What is the shape of the **B** field? [5]

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8. How is a plasma defined ? [2]

The *plasma frequency* is given as:

$$\omega_P = \sqrt{\frac{N_e e^2}{m_e \epsilon_0}}$$

Calculate the plasma frequency for a plasma of density $10^{12}\,\mathrm{m}^{-3}$. [2]

The dispersion relation for a plasma is:

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_P^2}{\omega^2}\right)$$

Briefly explain what happens to a wave of frequency ω entering a plasma from vacuum if:

(a) $\omega_P << \omega$ throughout the region; [2]

(b) $\omega_P > \omega$ in some part of the region. [2]

The Lorentz force on a charged particle is:

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Define the terms q, $\mathbf{E}$, $\mathbf{v}$ and $\mathbf{B}$ in the equation. [4]

Consider a plasma with a uniform external magnetic field (and no electric field) applied. First resolve the velocity of positively charged particles into components parallel to and perpendicular to the magnetic field, and write the components of the force on a particle in these directions. [4]

Then show that the perpendicular component leads to circulating motion with the following radius.

$$r = \frac{m v_\perp}{qB}$$ [4]

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9. The waveguide equation for $\mathrm{TE}_{\ell m}$ and $\mathrm{TM}_{\ell m}$ propagation modes in a guide with rectangular cross-section is $\dfrac{\ell^2}{a^2} + \dfrac{m^2}{b^2} = \dfrac{k_0^2 - k_g^2}{\pi^2}$. Define all of the quantities in this expression. [4]

Explain the meaning of the terms *TE* and *TM propagation modes.* [4]

There is a cut-off frequency for such a guide:

$$f_c = c\sqrt{\left(\frac{\ell}{2a}\right)^2 + \left(\frac{m}{2b}\right)^2}$$

What happens to signals in the guide below the cut-off frequency? Which values for the mode numbers ℓ and m are allowed for TE modes and which for TM modes? [4]

A rectangular waveguide has lateral dimensions 5 cm by 4 cm. Find the cut-off frequencies for the modes ℓm = 01, 10 and 11. State which of these modes can propagate along the guide at a frequency of 4.0 GHz, explaining your reasonng. [8]

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10. For a plane electromagnetic wave in a vacuum we can relate the magnitude of the wave vector to the angular frequency by: $k = \omega/c$. Write the equivalent relationship for a dielectric medium. [2]

The Poynting vector $\mathbf{N}$ is $\mathbf{E}\times\mathbf{H}$. Say briefly in words what this represents for a time-varying field. [2]

For an electromagnetic plane wave of the form $\mathbf{E} = \mathbf{E}_0 \exp i\,(\mathbf{k}\cdot\mathbf{r} - \omega t)$ in a medium with refractive index n and relative permeability $\mu_r = 1$ the magnitude of the Poynting vector can be written as:

$$N = \frac{1}{\mu_0}\frac{n}{c}E_0^2$$

Consider light incident on a dielectric surface, plane polarised with the electric field vector lying in the plane containing the incident and reflected wave vectors. Starting from the boundary conditions for $\mathbf{E}$ and $\mathbf{H}$ at a dielectric interface, show that the ratio of the amplitudes of the electric components of the incident and reflected waves is:

$$r_{\|} = \frac{n'\cos\alpha - n\cos\alpha'}{n'\cos\alpha + n\cos\alpha'}$$

where n and n' are the refractive indices of the incoming and outgoing sides of the interface and α and α' are the angles of incidence and of refraction. [6]

The reflected *intensity* coefficient is given by the ratio of the magnitudes of the Poynting vectors normal to the interface. Show that:

$$R_{\|} = r_{\|}^2$$

[4]

Consider a glass window pane with refractive index $n = 1.5$ (assume that air has $n = 1.0$). For incident light polarised in the plane containing the incident and reflected wave vectors, with an angle of incidence of 79°,

(a) Calculate the reflected intensity coefficient at the first (air/glass) interface. [4]

(b) Deduce from this the transmitted intensity coefficient at this interface. [2]

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11. The four Maxwell equations in differential form are

$$\begin{aligned}
\nabla \cdot \mathbf{D} &= \rho_f \\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\
\nabla \cdot \mathbf{B} &= 0 \\
\nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}
\end{aligned}$$

Name the observational law which each equation represents. [3]

Explaining all necessary assumptions, use the appropriate Maxwell equations to derive the wave equation

$$\nabla^2 \mathbf{E} - \sigma\mu \frac{\partial \mathbf{E}}{\partial t} - \epsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0.$$ [6]

Give the simplified form of this equation for waves in vacuum. Show that a solution to the equation in vacuum is a plane wave of the form $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp i(\mathbf{k} \cdot \mathbf{r} - \omega t + \phi)$, and hence show that the phase velocity of electromagnetic waves in vacuum is

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}.$$ [4]

Explain briefly, with the aid of appropriate sketches, what happens to the oscillations of the **E** vector in the following kinds of light:

plane polarised,
circularly polarised,
elliptically polarised,
unpolarised,
partially polarised. [7]

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