UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 2B29: Electromagnetic Theory

COURSE CODE : **PHYS2B29**

UNIT VALUE : **0.50**

DATE : **27-MAY-05**

TIME : **10.00**

TIME ALLOWED : **2 Hours 30 Minutes**

05-C1086-3-50

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Answer all SIX questions in SECTION A and THREE from SECTION B.
The numbers in square brackets at the right-hand side of the text indicate the provisional allocation of maximum marks per question or sub-section of a question.

You may need:

The Planck constant $h = 6.626 \times 10^{-34}$ Js.

Permeability of free space $4\pi \times 10^{-7}$ H m^{-1}.

Permittivity of free space 8.854×10^{-12} F m^{-1}.

Speed of light *in vacuo* 3×10^{8} m s^{-1}.

Displacement inside uniform parallel plate capacitor with area A and charge Q is given by $D = Q/A$.

$$\sqrt{i} = \frac{1}{\sqrt{2}}(1+i)$$

Theorems for any vector field **A**:

$$\nabla.\nabla \times \mathbf{A} = 0$$

$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla.\mathbf{A}) - \nabla^2 \mathbf{A}.$$

$$\oint_C \mathbf{A}.d\mathbf{l} = \int_S (\nabla \times \mathbf{A}).d\mathbf{s}$$

$$\oint_S \mathbf{A}.d\mathbf{s} = \int_V (\nabla.\mathbf{A})dv.$$

For a plane wave of the form $\mathbf{C}(\mathbf{r},t) = \mathbf{C}_0 \exp i(\mathbf{k}.\mathbf{r} - \omega t + \phi)$:

phase velocity $v_p = \dfrac{\omega}{k}$, group velocity $v_g = \dfrac{\partial \omega}{\partial k}$,

and $\dfrac{\partial \mathbf{C}(\mathbf{r},t)}{\partial t} = -i\omega \mathbf{C}(\mathbf{r},t)$; $\nabla.\mathbf{C}(\mathbf{r},t) = i\mathbf{k}.\mathbf{C}(\mathbf{r},t)$; $\nabla \times \mathbf{C}(\mathbf{r},t) = i\mathbf{k} \times \mathbf{C}(\mathbf{r},t)$.

SECTION A

1. Starting from the appropriate Maxwell equation, prove that the components of the electric field **E** parallel to a plane boundary between linear isotropic media have the same value on either side of the boundary. [6]

2. The Gauss law of electrostatics can be written in the form $\nabla.\mathbf{D} = \rho_f$.
Define the variables. Why is the equivalent equation for the divergence of the magnetic induction **B** simpler than the equation for the divergence of **D**? [3]

Use the appropriate mathematical theorem to transform the equation for divergence of **B** into an integral equation and explain briefly what this means. [3]

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3. Sketch the normal magnetization curve, the major hysteresis loop and a minor hysteresis loop for a typical ferromagnetic material. State briefly what each of them represents. Label the two axes and mark the point corresponding to the onset of saturation. Also mark the points at which coercivity H_c and remanence B_r are defined. State briefly what saturation, coercivity and remanence mean. [5]

Why is it impossible to give a unique number for the relative permeability of a ferromagnetic material? [2]

4. Starting from the appropriate Maxwell equation, prove that the components of the electrical displacement **D** perpendicular to a plane surface between linear isotropic media have the same values on either side of the surface, assuming there is no free charge on the surface. [7]

5. Use appropriate defining equations to derive the dimensions of at least seven of the following electromagnetic quantities in terms of the basic dimensions of mass [M], length [L], time [T] and charge [Q]:

Electric field strength **E**, potential difference V, capacitance C, dielectric constant κ, electric polarization **P**, electric induction **D**, permittivity of a medium ε, Magnetic field strength **H**, conductivity σ. [7]

6. Why is the retarded time $t' = t - \frac{r}{c}$ used to describe the electromagnetic fields at a distance r from an oscillating dipole? [3]

If the electrostatic potential due to such a dipole is given by the expression

$$\phi(\mathbf{r},t) \simeq \frac{q_0 l \cos\theta}{4\pi\varepsilon_0 r^2} \cos\omega t'$$

show how use of the retarded time gives rise to a wave moving outward in r. [4]

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SECTION B

7. If a plane electromagnetic wave in a linear isotropic medium has an electrical part $\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\exp i(\mathbf{k}.\mathbf{r}-\omega t)$ and a magnetic part $\mathbf{B}(\mathbf{r},t)=\mathbf{B}_0\exp i(\mathbf{k}.\mathbf{r}-\omega t)$, use appropriate Maxwell equations to demonstrate that $\mathbf{E}_0$, $\mathbf{B}_0$ and $\mathbf{k}$ are mutually perpendicular and that the moduli of the amplitudes satisfy the equation $kE_0=\omega B_0$.

The Poynting vector is $\mathbf{N}=\mathbf{E}\times\mathbf{H}$. Say briefly in words what this represents. Show that the average of the Poynting vector over a whole number of oscillation cycles is

$$<\mathbf{N}>=\frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}}E_0^2\,\hat{\mathbf{k}}$$ [10]

An unbounded plane wave in free space carries 10 watts of power per square metre. Calculate the size of the electrical amplitude E_0 and the magnetic amplitude H_0 in this wave, and give their units.

How many photons per second per square metre are required to carry this power if the frequency of the wave is 10 MHz? What pressure would it exert on a perfectly absorbing plane surface perpendicular to the wavevector **k**. [10]

8. The original integral form of the Ampère law can be written $\oint_P \mathbf{H}.d\mathbf{l}=\int_{S_P}\mathbf{J}_f.d\mathbf{S}$.

Explain, with sketches of the relevant surfaces of integration S_P, how this form of the law can give inconsistent results when applied to the current flowing into a parallel plate capacitor. Demonstrate that the integrals can be made consistent when the displacement current density term is added. [5]

Show that a plane wave of the form $\mathbf{H}(\mathbf{r},t)=\mathbf{H}_0\exp i(\mathbf{k}.\mathbf{r}-\omega t)$ which satisfies the wave equation $\nabla^2\mathbf{H}-\sigma\mu\frac{\partial\mathbf{H}}{\partial t}-\varepsilon\mu\frac{\partial^2\mathbf{H}}{\partial t^2}=0$ will also obey the dispersion relation

$$k^2=\mu\varepsilon\omega^2\left(1+\frac{i\sigma}{\varepsilon\omega}\right).$$ [5]

In a conducting medium, at a given angular frequency ω_m, it is found that the magnitude of the conduction current density **J(r,t)** due to a plane wave is much bigger than the magnitude of the associated displacement current density. What does this imply for the relative size of the two terms in brackets in the dispersion relation? Hence, how do you expect the wave to propagate through the medium? [6]

A medium has skin depth $50\mu m$ at 100 MHz. If its relative permeability is 1, what is its conductivity? [4]

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9. The Fresnel relations for light reflected and refracted at a plane uniform dielectric surface can be written as

$$t_{\perp} = \frac{2\cos\alpha}{\cos\alpha + (n_2/n_1)\cos\alpha'}$$

$$r_{\perp} = \frac{\cos\alpha - (n_2/n_1)\cos\alpha'}{\cos\alpha + (n_2/n_1)\cos\alpha'}$$

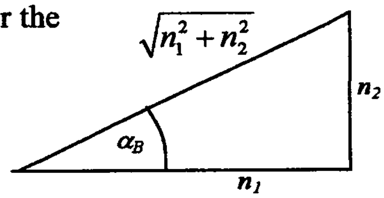
$$t_{\parallel} = \frac{2\cos\alpha}{(n_2/n_1)\cos\alpha + \cos\alpha'}$$

$$r_{\parallel} = \frac{(n_2/n_1)\cos\alpha - \cos\alpha'}{(n_2/n_1)\cos\alpha + \cos\alpha'}.$$

Draw a ray diagram and use it to define all of the quantities in these relations. [3]

What special behaviour occurs at the Brewster angle α_B? Use the appropriate Fresnel relation, together with Snell's law, to confirm that $\alpha_B = \tan^{-1} n_2/n_1$.

[Hint. It may help to consider the right angled triangle shown:



] [8]

Show that there can be a Brewster angle both when $n_1 < n_2$ and when $n_1 > n_2$. [3]

Unpolarised sunlight is reflected from a smooth lake surface. Assuming the refractive index of water is 1.33, what is the Brewster angle for the surface? A sailor sees an image of the Sun in the water at an angle of reflection equal to 60°. Calculate the factor by which the energy carried by this reflected light to his eyes is reduced when he puts on a pair of "perfect" polarising sunglasses with the optimal alignment. ("Perfect" implies complete transmission of light with one polarisation plane and complete absorption with the orthogonal plane). [6]

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10. The dispersion relation for waves in a plasma is $k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$, where $\omega_p = \sqrt{\frac{N_e e^2}{\varepsilon_0 m}}$. Define all of the quantities in these two expressions. Give a brief qualitative explanation of ω_p. [3]

Show that if $\omega < \omega_p$ electromagnetic waves will be absorbed as they pass through the plasma, with attenuation length $L = \frac{c}{\omega}\sqrt{\frac{1}{\left(\omega_p^2/\omega^2 - 1\right)}}$. [6]

A newly discovered planet is surrounded by a thin ionosphere. Exobiologists wish to study an intense source of 20 MHz radio waves on the surface of the planet, but it is severely attenuated every morning as the ionization density increases due to irradiation from the local star. Calculate the peak density N_{att} of free electrons in the plasma when the attenuation begins. [7]

A little later, when the attenuation has increased, only 10^{-6} of the power of the source can be observed. If the plasma layer is assumed to be uniformly ionised over a thickness of 10 metres, by what factor must the effective electron density have increased compared with N_{att}? [4]

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11. The waveguide equation for $TE_{\ell m}$ and $TM_{\ell m}$ modes in a guide with rectangular cross-section is $\frac{\ell^2}{a^2}+\frac{m^2}{b^2}=\frac{k_0^2-k_g^2}{\pi^2}$. Define all of the quantities in this expression.
There is a cutoff frequency for such a guide

$$\nu_c = c\sqrt{\left(\frac{\ell}{2a}\right)^2+\left(\frac{m}{2b}\right)^2}$$

What happens to signals in the guide below the cutoff frequency? Which values for the mode numbers ℓ and m are allowed for TE modes, and which for TM modes? [5]

A rectangular waveguide has lateral dimensions 5 cm by 9 cm. What range of frequencies of signals can be transmitted in it such that there are two and only two available TM and/or TE modes of transmission? [6]

By turning the waveguide equation into a dispersion relation show that the group and phase velocities in the guide satisfy the relations $v_p v_g = c^2$ and $v_p = \frac{c}{\sqrt{1-\left(\frac{\nu_c}{\nu}\right)^2}}$ [6]

Use this to work out the phase and group velocities of a 2 GHz signal in the given waveguide. [3]

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