UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 2B24: Atomic and Molecular Physics

COURSE CODE	:	**PHYS2B24**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05-C1083-3-120

TURN OVER

Answer ALL questions from Section A and THREE questions from Section B.

The following data may be used:
Speed of light, $c = 2.998 \times 10^8\,\mathrm{ms^{-1}}$
Planck's constant, $h = 6.626 \times 10^{-34}\,\mathrm{Js}$
Bohr magneton, $\mu_B = 9.274 \times 10^{-24}\,\mathrm{JT^{-1}}$
1 atomic unit of energy $= 219475\,\mathrm{cm^{-1}} = 27.2\,\mathrm{eV} = 2\,\mathrm{Rydberg}$
Fundamental electronic charge, $e = 1.602 \times 10^{-19}\,\mathrm{C}$
The numbers in square brackets show the provisional allocation of maximum marks per question or part of question.

Section A

1. State an expression which shows how the energy E_n of an energy level in the hydrogen atom varies with principal quantum number n in the simple Bohr model. (Use the Rydberg constant, R_∞, as the constant of proportionality.)

   From your expression deduce Rydberg's equation for the wavelength of the photon absorbed when an electon makes a transition from a state with quantum number m to one with quantum number n (where $m < n$), and so calculate the wavelength of the photon required to ionise the hydrogen atom. **[6]**

2. Discuss, with the aid of diagrams, the processes by which equilibrium is reached between a set of atoms, idealised to have just two energy levels each, and a black body radiation field inside a cavity. **[6]**

3. Explain how lasers may be used to cool atoms. **[8]**

4. Write down an expression for the interaction between an atom with electric dipole moment μ and an external electric field $\mathbf{E}_{ext}$, and hence distinguish between the linear and quadratic Stark effects. **[6]**

5. State Hund's Rules for the ordering of the energies of terms in a given electronic configuration.

   The electron configuration $2p^2$ has terms ^1S, ^3P and ^1D. Use the rules you have given to order them in energy from lowest to highest. **[7]**

6. If $\alpha(i)$ and $\beta(i)$ are the eigenstates of the operator $\hat{S}_z$ (the $z-$ component of spin) acting on electron number i, write down the symmetric and antisymmetric combinations of the spin wavefunctions of two electrons.

   What restriction does the Pauli Exclusion Principle place on the symmetry of the total wavefunction of a system of identical fermions?

   Since in the ground state of the helium atom the spatial part of the wavefunction must be symmetric, which of your above spin wavefunctions describes the ground state of helium? **[7]**

Section B

7. State, without proof, a formula for the energy levels, E_{nl}, of an alkali metal atom in terms of an effective nuclear charge Z_{eff}, the principal quantum number n, and the quantum defect Δ_{nl}. [3]

   Explain and qualitatively justify how Δ_{nl} varies with the quantum numbers n (principal) and l (orbital angular momentum). [5]

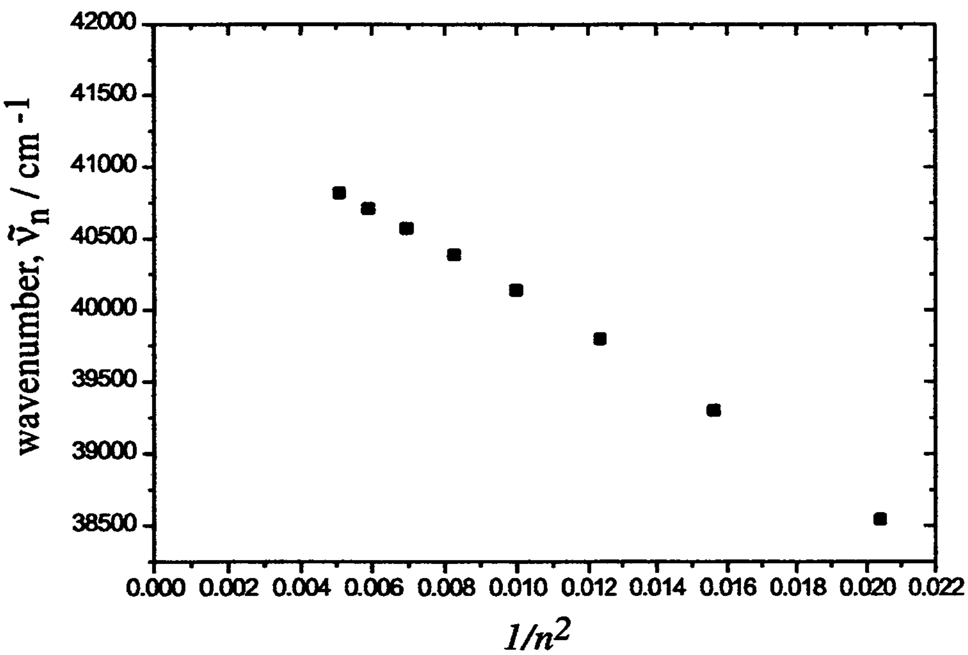
   The table below contains eight of the wavenumbers $\tilde{\nu}_n$ for the transitions 3S - nP in sodium, and the data is plotted in the accompanying graph as a function of $1/n^2$.

   Explain the physical significance of the intercept of the graph with the vertical ($\tilde{\nu}_n$-) axis. [4]

   Use either the tabulated data or the graph to estimate the first ionisation potential of sodium, both in wavenumbers and in electron volts, and hence the quantum defect of the 3S level. [8]

n	7	8	9	10
$\tilde{\nu}_n$ / cm^{-1}	38,541	39,299	39,795	40,137

n	11	12	13	14
$\tilde{\nu}_n$ / cm^{-1}	40,383	40,566	40,706	40,814



8. Describe and explain the main features of the X-ray spectrum produced when an energetic electron beam strikes a target. Discuss qualitatively how the spectrum changes as the electron energy is increased, and when a different target material is used. [8]

By assuming the states of electrons in heavy atoms are hydrogen-like, obtain a simple relationship between the wavelength of X-ray K_α lines and the atomic number Z of the target material. [4]

The data below comes from an experiment to measure the wavelength of X-ray K_α lines from a variety of targets. Use the data to obtain a mean value for the Rydberg constant, R_∞. [8]

Material	$\lambda(K_\alpha)$ / pm
$_{4}$Be	11,374.4
$_{10}$Ne	1,462.0
$_{20}$Ca	335.8
$_{24}$Cr	229.1
$_{30}$Zn	143.5
$_{42}$Mo	71.1
$_{50}$Sn	49.1

9. Distinguish between elastic and inelastic collisions between an electron and an atom. [2]

Explain how the Franck-Hertz experiment provides evidence for the quantisation of electronic energy levels in an atom. [7]

In a Franck-Hertz type experiment, atomic hydrogen is bombarded by electrons that have been accelerated through a potential V, and the rate of arrival of electrons at the anode measured as a current I. It is found that the current falls sharply when the potential V has the values 10.2 V and 12.1 V.

What are the principle quantum numbers of the states to which the hydrogen atoms have been excited? [4]

If the light emitted in the decay of the excited hydrogen atoms were to be analysed, how many spectral lines in total would be observed as the potential is increased from 0 V to 12.1 V? Identify the spectral series to which the lines belong. [4]

If the potential were increased further to 12.6 V how many more spectral lines would be observed? Justify your answer. [3]

10. The Hamiltonian for an alkali metal atom in a magnetic field **B** is

$$H = H_0 + H_{LS} + H_B,$$

where H_0 is the unperturbed Hamiltonian and H_{LS} and H_B are small corrections due to the spin-orbit interaction and the Zeeman effect respectively.

Explain the origins of H_{LS} and H_B in terms of the interaction between a magnetic dipole moment and a magnetic field.

What effect does each of these have on the energy levels of the atom?

Comment on the limiting cases $H_{LS} \gg H_B$ and $H_{LS} \ll H_B$. [7]

With the aid of vector diagrams, illustrate the coupling of spin (**S**) and orbital (**L**) angular momentum in the limits above, for each case stating the good quantum numbers. [4]

Given that in zero magnetic field the highest and lowest energy levels of the $(3s^1 3p^1)\,^3P$ first excited state of magnesium are separated by $60\,\text{cm}^{-1}$, calculate the separations of all the components of $(3s^1 3p^1)\,^3P$. (You may if you wish assume the Landé interval rule without proof.) [5]

Derive a criterion for the magnitude of external magnetic field that would be considered strong in this case. [4]

11. The energy of an idealised diatomic molecule may be approximated as

$$E \simeq E_{electron} + BJ(J+1) + \left(\nu + \frac{1}{2}\right)\hbar\omega.$$

Explain what each of the terms in the above expression are, and define the symbols B, J, ν and ω. [4]

Under what conditions is this approximation valid? [3]

The results of an infra-red absorption spectroscopy experiment on carbon monoxide show a band centred on $2167.9\,\text{cm}^{-1}$, and a line spacing in the region of the band centre of approximately $3.86\,\text{cm}^{-1}$.

Use this information to calculate the parameter B that appears in the above equation. Calculate also the moment of inertia and the reduced mass of the molecule, and the spring constant of the bond. [8]

Sketch the appearance of the infra-red absorption spectrum of CO indicating any deviations from the idealised case considered and explain their cause. [5]

[The relative atomic masses of carbon and oxygen may be taken as 12 and 16 respectively, and the mass of a nucleon as 1.67×10^{-27} kg.]

END OF PAPER