UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

Physics 1B72: Waves and Modern Physics

COURSE CODE	:	**PHYS1B72**
UNIT VALUE	:	**0.50**
DATE	:	**22-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

06-C1163-3-40

TURN OVER

Answer ALL questions from Section A and THREE questions from Section B.

For each sub-section of a question, the provisional allocation of marks is indicated in square brackets.

Elementary charge, $e = 1.60 \times 10^{-19}$ C
Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Planck's constant, $h = 6.63 \times 10^{-34}$ J s
Permittivity of vacuum, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Speed of light in vacuum, $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$

SECTION A

1. A mass on a spring oscillates in simple harmonic motion, moving up and down over a total distance of 60 mm. The period of motion is T = 2 s. What are the values (and units) of the amplitude and angular frequency? [2]

A clock is started at time t = 0 s, when the mass is at its mimimum height above the floor. Calculate the position and velocity of the mass at t = 8.5 s. [5]

2. Using the Lorentz transformation equations,

$$x' = \gamma(x - vt)\,,$$
$$y' = y\,,$$
$$z' = z\,,$$
$$t' = \gamma(t - vx/c^2)\,,$$

show that

$$u'_x = \frac{(u_x - v)}{(1 - u_x v/c^2)}\,.$$

u_x is the $x-$component of velocity in the inertial frame of reference S and u'_x is the $x-$component of velocity in the inertial frame S'. Frame S' is moving relative to frame S with uniform speed v in the $x-$direction. [4]

Use the above result to show that photons move with speed c in ALL inertial frames. [2]

3. Give a physical example of (a) longitudinal, and (b) transverse wave motion. Describe a physical property that may be used to distinguish transverse waves from longitudinal waves. [3]

Derive a general expression for a one-dimensional wave travelling with constant speed v and without change of shape, along the positive direction of the x-axis. [4]

4. State Rayleigh's criterion for the resolution of two point sources of light and deduce their angular separation when the criterion is satisfied. [4]

When Mars is closest to the Earth, it is at a distance of 88.6×10^6 km away. If Mars is viewed with a telescope having a mirror diameter of 60 cm, use the Rayleigh criterion to calculate the angular resolution of the telescope, for a wavelength of 550 nm. Hence calculate the smallest distance between two points that can be resolved on the surface of Mars. [3]

5. In a study of the photoelectric effect, a potassium metal surface is illuminated with light whose wavelength is 500 nm. The work function of potassium metal is 2.22 eV. Calculate the maximum kinetic energy (in eV) of the photoelectrons. [4]

Explain the meaning of the phrase *cut-off frequency*, as applied to the photoelectric effect. [3]

6. A mobile phone emits 150 mW of electromagnetic radiation at a frequency of 1750 MHz. Calculate:

(a) the energy of each photon, and [2]

(b) the number of photons emitted per second. [2]

If one could make a laser, operating at a wavelength of 550 nm, which emitted as many photons per second as does the above mobile phone, what would be the power output of such a laser? [2]

SECTION B

7. State the Uncertainty Principle for position and momentum and explain briefly its meaning. [4]

A particle confined in a box of length L lying along the x−axis has an uncertainty in its position of $\Delta x = L$. Use the Uncertainty Principle to estimate its ground state energy. [6]

Calculate the exact value of the ground state energy and compare this with the value obtained using the Uncertainty Principle. [10]

8. Light of wavelength λ is incident normally on a diffraction grating with slit spacing d. Derive the condition for constructive interference at the angle θ. [5]
Give the allowed values of the order number, commenting on the directions to which they correspond. [2]

A diffraction grating has 52 500 lines spread over a width of 42 mm. Calculate the angular width in the first-order spectrum between the sodium D-lines with wavelengths $\lambda_1 = 588.995\,\text{nm}$ and $\lambda_2 = 589.592\,\text{nm}$. [4]
Comment on your result. [1]

Define the resolving power R of a grating and write down, without proof, a simple expression for R in the mth order diffraction if N lines of the grating are illuminated. [4]

What is the minimum number of lines that a diffraction grating must have in order to resolve the sodium D-lines in second-order? [4]

9. State the two postulates of Einstein's theory of special relativity. [4]

The inertial frame S' moves along the positive x−axis with speed v relative to S. A rod at rest in S', has its length (in the direction of motion) measured as L_0 by an observer in S', and as L_v by an observer in S. Write down the expression for L_v in terms of L_0. [3]

A metre stick oriented to lie along the direction of motion approaches an observer with speed $\frac{24}{25}c$. What is the length of the stick in the observer's frame? [2]

Suppose instead that the metre stick is oriented at an angle to the x−axis of

$$\theta' = \cos^{-1}(5/6)$$

in the frame moving with the stick. In the frame of the observer, calculate the length of the stick [6]
and the angle of orientation θ. [5]

10. Using the concept of de Broglie waves, explain the criterion that allows only certain orbits for the electron in a hydrogen atom. [2]
Obtain Bohr's angular momentum quantization condition

$$m_e v r = n\hbar \,,$$

where v is the speed of the electron (mass m_e) in orbit of radius r. [4]
What is the value of n corresponding to the ground state? [1]

Use Bohr's model to obtain an expression for r_1, the radius of the electron's orbit in the ground state. [7]
Show that the corresponding orbital speed of the electron is

$$v_1 = e^2/2\epsilon_0 h \,. \quad [2]$$

Calculate v_1 and by comparing this value with the speed of light, consider whether it is a reasonable approximation to neglect relativistic effects in the theory of the hydrogen atom. [4]

11. Show that the frequency f_o measured by an observer who is approaching, at a constant speed v_o, a stationary source which emits sound at a constant frequency f_s, is

$$f_o = f_s\left(1 + \frac{v_o}{v}\right) \,,$$

where v is the speed of sound. [5]

Write down the general Doppler formula for f_o if the source is moving with speed v_s, the observer with speed v_0 and the medium with speed v_m. Assume that **all** speeds are measured relative to the ground in the positive direction of the x−axis. [3]
If the source and observer are at rest, how are f_o and f_s related? Comment on your result. [2]

A train is moving parallel to a motorway at $20\,\mathrm{ms^{-1}}$. A car is travelling in the same direction as the train at $30\,\mathrm{ms^{-1}}$. The car horn sounds at 555 Hz and the train whistle at 360 Hz. The speed of sound in air is $340\,\mathrm{ms^{-1}}$.

(a) When the car is behind the train, what frequency does the driver of the car observe for the train's whistle? [5]

(b) When the car is in front of the train, what frequency does a passenger on the train observe for the car's horn? [5]