UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

Physics 1B72: Waves and Modern Physics

COURSE CODE	:	**PHYS1B72**
UNIT VALUE	:	**0.50**
DATE	:	**24-MAY-05**
TIME	:	**14.30**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05-C1079-3-50

TURN OVER

Answer ALL questions from Section A and THREE questions from Section B.

For each sub-section of a question, the provisional allocation of marks is indicated in square brackets.

Elementary charge, $e = 1.60 \times 10^{-19}$ C
Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Planck's constant, $h = 6.63 \times 10^{-34}$ J s
Speed of light in vacuum, $c = 3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$

SECTION A

1. A particle of mass m moves under the influence of a restoring force proportional to its displacement, $F = -kx$. What type of motion does the particle exhibit? [1]

Write down an expression for the particle's displacement $x(t)$ explaining the meaning of each term in the equation you write. [2]

By substituting in Newton's second law of motion, derive an expression for the angular frequency of the particle's motion. [3]

Suppose a particle undergoes such motion with amplitude 1.5 cm. What is the total distance that it travels in one period? [1]

2. The relativistic kinetic energy of a particle with proper mass m and speed u is

$$K = (\gamma - 1)mc^2$$

where

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}.$$

Give an expression for the relativistic momentum of the particle. [1]

Show that K approaches the Newtonian expression for the kinetic energy when $u \ll c$. [3]

At what speed is the kinetic energy of a particle equal to its rest energy? [2]

3. Consider the two disturbances below,

$$\psi_1 = 2\sin(2x + t) + 3\cos(6x + 3t)\,,$$
$$\psi_2 = 2\cos x\ \cos 2t\,.$$

Which of these is a travelling wave in one dimension, and why? What is the speed of this wave and in which direction is it travelling? [4]

What name is given to the other disturbance, and why? [3]

4. State Rayleigh's criterion for the resolution of two point sources of light and deduce their angular separation when the criterion is satisfied. [4]

The primary mirror of a space telescope that orbits 500 km above the Earth, has a diameter of 2.0 m. Using the Rayleigh criterion, calculate the separation of the most closely spaced objects that it could resolve when viewing the Earth's surface. Assume light of wavelength $\lambda = 550\,\text{nm}$. [3]

5. State Heisenberg's uncertainty principle, explaining briefly the meaning of the symbols used. [2]

A free electron has a kinetic energy of 100 eV. Calculate its momentum in the non-relativistic approximation. [2]

Its speed can be measured with 1% accuracy. What is the minimum uncertainty in its position? [3]

6. A radio station broadcasts at a frequency of 10 MHz with a total radiated power of 10 000 W. Calculate:

(a) the wavelength of the radiation, [2]

(b) the energy(in eV) of each photon that comprises the radiation, [2]

(c) the number of photons emitted per second, [2]

SECTION B

7. Write down the de Broglie equation relating the wavelength of a moving particle to its momentum. [2]

What is varying in the case of matter waves and how is it related to experiment? [3]

Calculate the kinetic energy (in joules) of an electron which has been accelerated through an electrostatic potential difference of 5 kV. [2]

What is the value of the momentum of the electron, and its SI units? [3]

Use the de Broglie formula to calculate the wavelength of the electron in nm. [2]

Calculate the energy of a photon having this wavelength. [2]

In which region of the electromagnetic spectrum is this photon? [1]

A beam of such electrons undergoes first-order Bragg reflection from a set of parallel crystal planes which are 2×10^{-10}m apart. State Bragg's law and use it to find the angle between the direction of the incident beam and the scattered beam. [5]

8. A monochromatic light beam in the form of a plane wave is incident normally on a long, narrow, horizontal slit of width a (in the vertical direction). A Fraunhofer diffraction pattern of alternating dark and bright fringes is observed on a distant screen. Calculate the angular positions of the diffraction minima. [12]

Show that the central bright fringe is twice the width of the subsidiary bright fringes. [4]

Given that around 85% of the power in the transmitted beam is in the central bright fringe, sketch, roughly to scale, the intensity distribution I as a function of the parameter $z = \frac{\pi}{\lambda} a \sin\theta$. [4]

9. State the two postulates of Einstein's theory of special relativity. [4]

Two Cartesian frames of reference, S and S', are coincident at time $t = 0$. For $t > 0$, S' moves with speed v, along the positive $x-$axis, with respect to S. In S', an observer measures the time interval between two events, which occur at the same location, as Δt_0. What is this time called? [1]

An observer in S measures the dilated time interval Δt_v between the same two events. Derive, with the aid of a thought experiment, the relation

$$\Delta t_v = \Delta t_0 / \sqrt{1 - \frac{v^2}{c^2}}\,.$$ [10]

An elementary particle is known to have a lifetime of 2×10^{-10} s in its own rest frame. If it is seen by an inertial observer to have a speed of 0.99 c, how far will the observer say that it travels before decay? [5]

10. Explain why the wavefunctions for a particle of mass m trapped within an infinite square well potential extending from $x = 0$ to $x = L$ *must* be zero at $x = 0$ and $x = L$. [3]

If the wavefunctions are given by:

$$\psi_n(x) = A \sin(\frac{n\pi x}{L}) \qquad (n = 1, 2, 3, \ldots)$$

(a) Deduce an expression for the energy levels of the particle. [5]

(b) Calculate the normalisation constant A. [5]

(c) Sketch the wavefunction and probability density for the lowest energy state. [3]

(d) By considering your sketch, or otherwise, calculate the probability of finding the particle in the region $L/2 \leq x \leq L$, if it is in its lowest energy state. Write down an integral giving the probability that the particle is in the region between $x = 0$ and $x = L/4$. [4]

Note: $\cos 2\alpha = 1 - 2\sin^2\alpha\,.$

11. State the principle of superposition of waves. [2]

Consider two waves of slightly different angular wavenumber and angular frequency but of the same amplitude, namely

$$y_1 = A\ \cos(k_1 x - \omega_1 t)\,,$$

$$y_2 = A\ \cos(k_2 x - \omega_2 t)\,.$$

Show that the resultant disturbance arising from the combination of the two waves is given by

$$y = 2A\ \cos(\bar{k}x - \bar{\omega}t)\ \cos(k_m x - \omega_m t)\,,$$

and derive expressions for the terms $\bar{k}, \bar{\omega}, k_m$ and ω_m. [6]

Sketch the form of the resultant disturbance. [3]

State the mathematical expression that defines the group velocity v_g . [2]

Show that v_g and the phase velocity v are related by

$$v_{\rm g} = v + k\frac{\mathrm{d}v}{\mathrm{d}k}\,.$$ [3]

Suppose that the dispersion relation for waves on a thin elastic plate is $\omega = Ak^3$, where A is a constant. Find the relationship between the two velocities for these waves. [4]

Note: $\cos\alpha + \cos\beta = 2\ \cos\left(\frac{\alpha+\beta}{2}\right)\ \cos\left(\frac{\alpha-\beta}{2}\right).$