UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

Physics 1B72: Waves and Modern Physics

COURSE CODE : **PHYS1B72**

UNIT VALUE : **0.50**

DATE : **10-MAY-04**

TIME : **10.00**

TIME ALLOWED : **2 Hours 30 Minutes**

04-C1080-3-50

TURN OVER

Answer ALL questions from Section A and THREE questions from Section B.

For each sub-section of a question, the provisional allocation of marks is indicated in square brackets.

Elementary charge, $e = 1.60 \times 10^{-19}$ C
Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg
Planck's constant, $h = 6.63 \times 10^{-34}$ J s
Permittivity of free space, $\varepsilon_0 = 8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Speed of light in vacuum, $c = 3.00 \times 10^8\,\mathrm{m\,s^{-1}}$

SECTION A

1. Write down the differential equation that defines simple harmonic motion (SHM) and explain the physical significance of any constants appearing in the equation. [2]

A spot of light on the screen of a cathode-ray tube is simultaneously subject to SHM in the $x-$ and $y-$directions, with displacements

$$x = a\cos\omega t$$

and

$$y = b\cos(\omega t - \phi).$$

If the light spot follows a circular path in the $xy-$plane when $\phi = \pi/2$, determine the relationship between the amplitudes a and b. [4]

Note: You may find the following relation useful,

$$\cos(A-B) = \cos A\cos B + \sin A\sin B.$$

2. In Special Relativity, a particle has proper mass m, speed u and rest energy E_0. The total energy E and momentum p of the particle are given by

$$E = \gamma E_0 \qquad \text{and} \qquad p = \gamma m u,$$

where

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}.$$

Derive the energy-momentum relation

$$E^2 = {E_0}^2 + p^2c^2.$$

[7]

3. Show that the frequency f_o measured by an observer who is moving away, at a speed v_o, from a stationary source which emits sound at a constant frequency f_s is

$$f_o = f_s \left(1 - \frac{v_o}{v}\right) ,$$

where v is the speed of sound. [3]

State the full expression describing the situation when the source *and* the observer move in opposite directions at speeds (relative to the ground) of v_s and v_o, respectively. [2]

A stationary source emits a steady note at a frequency of 1500 Hz. Calculate the frequency that will be heard by an observer moving away from the source at $34\,\mathrm{m\,s^{-1}}$. Assume that the speed of sound in air is $340\,\mathrm{m\,s^{-1}}$. [2]

4. State Rayleigh's criterion for the resolution of two point sources of light and deduce their angular separation when the criterion is satisfied. [3]

What is the minimum distance that can be resolved (with light of wavelength $\lambda = 600\,\mathrm{nm}$) on the surface of the Moon with a telescope whose lens has a diameter of 15 cm? Assume that the distance between the Earth and the Moon is 400,000 km. [4]

5. State the energy-time uncertainty principle and explain the meaning of the symbols used. [2]

A discharge tube produces an emission line of wavelength 500 nm with an intrinsic width of 2×10^{-4} nm. Assuming that the transition is from an excited state to the ground state, estimate the life-time of the excited state. [5]

6. The radiation emitted by a He-Ne laser has wavelength $\lambda = 633\,\mathrm{nm}$. If the laser has a power of $0.5\,\mathrm{mW}$, calculate:

(a) the energy of each photon, [2]

(b) the momentum of each photon, [2]

(c) the number of photons emitted per second. [2]

SECTION B

7. Outline one experiment that provides evidence for the wave-like properties of electrons, explaining how the wavelength of the electrons may be deduced from the experimental measurements. [12]

State the de Broglie relation for the wavelength of a moving particle in terms of its momentum. Give an expression for the frequency of the associated matter wave in terms of the total energy of the particle. Hence, calculate the phase velocity of a de Broglie wave and show that it is faster than c, the speed of light in vacuum. [6]

Give a formula for calculating the group velocity of a moving body. With what speed would you expect the wave group associated with a moving body to travel. (An explicit calculation is NOT required.) [2]

8. In a Young's double-slit experiment, monochromatic light with wavelength λ illuminates two narrow slits separated by a distance d. Fringes are observed on a screen at a distance $D \gg d$ from the plane of the slits. Calculate the path difference between light arriving at a position on the screen from the two slits. Show that the separation between neighbouring bright fringes is $\lambda D/d$. Say whether the central fringe is bright or dark and explain your reason. [9]

If $d = 0.2\,\text{mm}, D = 80\,\text{cm}$ and neighbouring bright fringes are 2 mm apart, calculate the wavelength of the light in nm. [4]

Explain the historical importance of Young's experiment in understanding the nature of light. Define the concept of *coherence*, and use it to explain why interference effects would NOT have been observed using two separate light sources. [7]

9. State the two postulates of Einstein's theory of special relativity. [4]

Consider two inertial frames of reference, S and S'. The frame S' moves along the $x-$axis with speed v relative to S. Stationary observers are situated in both frames and the observer in S has a clock, which can be seen by both observers. A rod, at rest in S', has its length (in the direction of motion) measured as L_0 by the observer in S', and as L_v by the observer in S. With the aid of a *thought experiment*, derive the relation

$$L_v = L_0\sqrt{1 - \frac{v^2}{c^2}}\,.$$

You may assume the time dilation formula. [12]

How much time does a metre rod, moving at $0.8c$ relative to an observer, take to pass that observer? Assume that the metre rod is parallel to its direction of motion. [4]

10. Use Bohr's model to deduce the energy levels of the hydrogen atom:

$$E_n = -\frac{m_e e^4}{8\epsilon_o{}^2 h^2 n^2} \qquad (n = 1, 2, 3, \ldots)$$

[12]

Show that the Balmer series of wavelengths is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \qquad (n = 3, 4, 5, \ldots)$$

and calculate the value of R in nm^{-1}. [5]

Why are spectral lines corresponding to very large values of n rarely seen in nature? [3]
[You may wish to use the numerical result $E_n = -13.6/n^2$ eV .]

11. Give a physical example of (a) longitudinal, and (b) transverse wave motion. Describe the physical feature that distinguishes transverse and longitudinal waves. [4]

Derive a general expression for a one-dimensional transverse wave travelling with constant speed v and without change of shape, along the positive direction of the x–axis. Write down the corresponding expression for a similar wave travelling in the opposite direction. [4]

State the second-order partial differential equation that describes all types of wave motion in one dimension. Show that both the travelling waves mentioned above are solutions of this equation. [6]

Which of the disturbances, (i) - (iii) below, represent a travelling wave in one dimension?

(i) $\psi = e^{ikx}e^{i\omega t}$

(ii) $\psi = \sin 2x \cos 3t$

(iii) $\psi = e^{-\alpha(3x-t)^2}$

In each case, what is the speed of the wave and in which direction is it travelling? If any of the three disturbances does NOT represent a travelling wave, then explain what it *does* represent. [6]