UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:–

B.Sc. *M.Sci.*

Physics 1B26: Electricity and Magnetism

COURSE CODE	:	**PHYS1B26**
UNIT VALUE	:	**0.50**
DATE	:	**10–MAY–05**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05–C1072–3–30

TURN OVER

Answer *ALL* SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

SECTION A

[Part marks]

1. (a) State Coulomb's law for two point charges q_1 and q_2 separated by a distance r_{21}. Define the constants appearing in Coulomb's law. [2]

   (b) Write down the expression for the magnetic force acting on a charged particle (charge q) moving at a velocity $\underline{v}$ in a magnetic field $\underline{B}$. Explain why this force cannot alter the particle's kinetic energy. [4]

2. (a) Define the electric field $\underline{E}$. [2]

   (b) Explain how the electric field lines are related to the direction of the electric field. [2]

   (c) Describe, and sketch, the electric field lines for:

   i. a point positive charge [1]

   ii. a point negative charge [1]

   iii. an electric dipole [1]

3. (a) State the principle of superposition. [3]

   (b) By using this principle, write down the electric field due to a collection of point charges q_1, q_2.... q_n. [3]

4. (a) State Gauss' law for electrostatics in integral form. [2]

   (b) By using this law determine the electric field E outside of an insulating sphere of radius R uniformly charged with total charge Q. [4]

5. (a) Define the capacitance of a pair of conductors. [3]

   (b) Determine the capacitance of a parallel plate capacitor. [4]

6. (a) Define the electric current I. [2]

(b) Assuming that the current is produced by the motion of particles with charge q moving on average at a drift velocity $\underline{v}_d$, write down the vector current density $\underline{J}$ in terms of q, $\underline{v}_d$, and the charged particles density n. Relate $\underline{J}$ to the electric current I. [2]

(c) State Ohm's law in terms of the current density $\underline{J}$. [2]

(d) State the expression for the equivalent resistance R_{eq} of two resistors R_1, R_2 in series. [1]

(e) State the expression for the equivalent resistance R_{eq} of two resistors R_1, R_2 in parallel. [1]

SECTION B

7. The points A,B,C are the vertices of an equilateral triangle, of side L. Two equal positive charges q are at A and B.

(a) Find the electric potential at C, assuming $V = 0$ at infinity. [3]

(b) Give the expression for the work necessary to bring a positive charge Q from infinity to C. [3]

(c) Write the expression for the work necessary to assemble the system of three charges, consisting of two charges q at A and B, and a charge Q at C. [4]

(d) Consider the system of three charges. By differentiating the electric potential, determine the electric field $\underline{E}$ at the midpoint between the two charges q. [4]

Consider now a system of two charges: a point charge $q > 0$ located at the position $(x, y, z) = (a, 0, 0)$ and a point charge $-q/2$ located at $(-a, 0, 0)$. Show that the equipotential surface $V = 0$, i.e. with the same potential than at infinity, is a spherical surface. Determine the centre and the radius of the sphere. [6]

8. (a) A spherical surface of infinitesimal thickness and radius R_1 is uniformly charged. The total charge distributed on the surface is q_1. Determine the electric field both for $r < R_1$ and $r > R_1$. [3]

(b) Consider now a system of two concentric spherical surfaces, with radii R_1,R_2 ($R_2 > R_1$) and charges q_1,q_2. Determine the electric field in the regions $r < R_1$, $R_1 < r < R_2$, $R_2 < r$. [5]

(c) Consider once again the system described in (b). Determine the ratio and relative sign of q_1 and q_2 so that $E(r) = 0$ for $r > R_2$. [4]

(d) Consider now the case of two spherical surfaces with radii R_1,R_2 ($R_2 > R_1$) and charges q_1,q_2. The centres O_1, O_2 of the two spheres are separated by a small distance a ($a << R_1$; $a < R_2 - R_1$). The midpoint of O_1O_2 will be denoted by O, and the distance of P from O by r.

Give the exact expression for the electric potential V(P) at a point P far away from the spheres. [1]

By expanding $V(P)$ in powers of a/r, with $r >> R_2$ the distance of P from the midpoint of O_1O_2, determine the leading term in the potential in the two cases:

i. $q_1 + q_2 \neq 0$ [3]

ii. $q_1 + q_2 = 0$. [4]

9. (a) Write the general expression for Biot-Savart law, defining the quantities involved. [3]

(b) Show that the magnetic field on the axis of a circular current loop of radius R is

$$\underline{B} = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \hat{x}$$

where the x axis coincides with the loop axis, and $x = 0$ at the centre of the loop. [7]

(c) Two parallel circular current loops, of radius R, are at a distance d and share the same axis. The magnitude of the current in the two loops is the same, but the direction is opposite. Determine the gradient of the magnetic field on the axis of the system of two coils, halfway between them. [5]

(d) Consider an insulating circle of radius R carrying a total charge Q uniformly distributed. The circle is rotating around its axis at the angular velocity ω. Determine the magnetic field B at the center of the loop. [5]

10. (a) Define the magnetic flux. [2]

(b) State and explain Faraday's law. [2]

(c) A metallic rod of length l rotates at the angular velocity ω around an axis passing through one end of the rod perpendicular to the rod. A constant and uniform magnetic field B_0 is applied. Determine the EMF induced between the two ends of the rod on the two cases:

i. B_0 perpendicular to the rotation axis. [1]

ii. B_0 parallel to the rotation axis. [4]

(d) State Ampère's law. [2]

(e) By using Ampère's law show that the magnetic field produced outside an infinitely long straight conducting wire currying a current I is $B = \mu_0 I/(2\pi r)$. [4]

(f) An infinitely long straight wire, placed along the y axis, carries a current I. A rectangular loop of wire, of sides a and b, moves in the xy plane at constant velocity (in the x direction) $\underline{v} = v\underline{u_x}$, with the sides of length b parallel to the y axis. The whole loop starts on the side $x > 0$ of the wire and does not cross it. If at time $t = 0$ the side of the loop closer to the y axis is at distance x_0 from it, show that the EMF induced in the loop is

$$\mathcal{E} = \frac{\mu_0 I a b v}{2\pi x(x + a)}$$

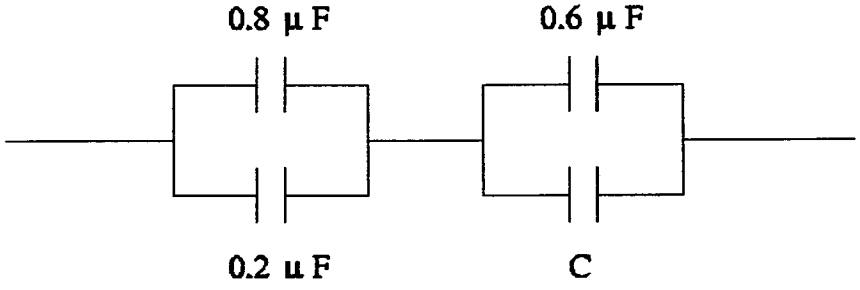
with $x = x_0 + vt$. [5]

11. (a) Write down the expression for the equivalent capacitance C_{eq} of two capacitors C_1, C_2

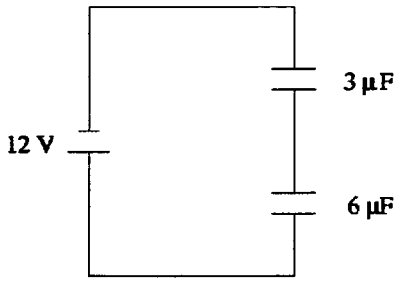
i. in parallel [1]

ii. in series [1]

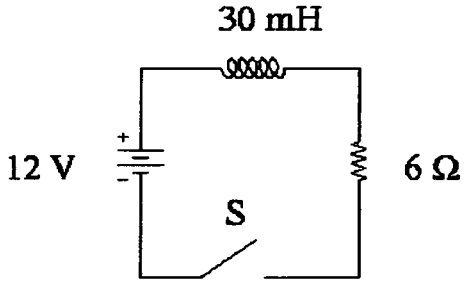
(b) Consider the system of four capacitors shown in the picture. For which value of the capacitance C does the system have an equivalent capacitance $C_{eq} = 0.5\mu$F? [3]



(c) Consider the system of two capacitors shown in the picture. Determine the magnitude of the voltage across each capacitor. [3]



(d) Consider the series circuit composed of a switch, and inductor, a resistor and a battery (see Figure). The switch S in the figure was open for a very long time and it is closed at time $t = 0$.

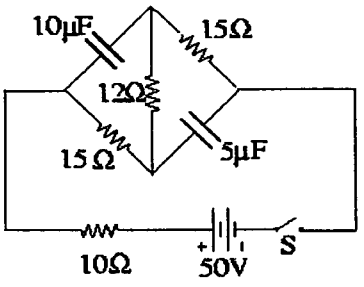


i. Find the time constant τ of the system [2]

ii. Calculate the current in the circuit at $t = 2$ ms [3]

iii. Find the voltage across the resistor after one time constant $t = \tau$ [3]

(e) The capacitors in the circuit below are initially uncharged. What is the current through the battery a long time after the switch S is closed? [4]



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