UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Physics 1B26: Electricity and Magnetism

COURSE CODE	:	**PHYS1B26**
UNIT VALUE	:	**0.50**
DATE	:	**25-MAY-04**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

04-C1072-3-70

TURN OVER

Answer _ALL_ SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

electronic charge,	e	$= 1.60 \times 10^{-19}$ C
permittivity of free space,	ϵ_0	$= 8.85 \times 10^{-12}$ $\mathrm{Fm^{-1}}$
permeability of free space,	μ_0	$= 4\pi \times 10^{-7}$ $\mathrm{TmA^{-1}}$

SECTION A

[Part marks]

1. An electric dipole consists of two charges $+q$ and $-q$ in the xy plane at the positions $(-d, 0)$ and $(d, 0)$ respectively.

   (a) Sketch the electric field lines and equipotentials associated with this dipole in the xy plane. **[3]**

   (b) Determine the magnitude and direction of the electric field $\underline{E}$ at the point $(0, d)$. **[4]**

2. Write down the Lorentz force acting on a particle of charge q, velocity $\underline{v}$ in a region of electric field $\underline{E}$ and magnetic field $\underline{B}$. **[2]**

   Explain, with a diagram, how the simultaneous application of an electric and magnetic field can be used to select charged particles of a particular velocity. **[4]**

3. Derive an expression for the current flowing in a conductor of cross-sectional area A in terms of the number density, charge and velocity of the particles that carry the charge. **[3]**

   Describe, with the aid of a diagram, the path of a charge carrier through the conductor. **[3]**

4. State the Kirchhoff rules for electrical networks. **[2]**

   Use the rules to establish the overall resistance of two resistors R_1 and R_2 connected in

   (a) series **[2]**

   (b) parallel **[3]**

5. Describe and explain the arrangement of charges within, and on the surface of, an irregularly shaped charged conductor in electrostatic equilibrium. Also, discuss the electric field and electric potential within the conductor. **[4]**

   Use the method of image charges to determine the electrostatic force acting upon a charge Q located a distance d above an infinite earthed conducting plate. **[3]**

6. State Ampère's law for steady currents. [2]

Using the example of charging a parallel-plate capacitor, show how this law is inadequate for a time-varying current. [3]

State how Ampère's law can be modified to correctly describe a time-varying current. [2]

SECTION B

7. Write down a general expression for the electrostatic energy of a set of n charges, $q_1, q_2, \ldots, q_n$ at positions $\underline{r}_1, \underline{r}_2, \ldots, \underline{r}_n$ respectively, and a specific expression for the case $n = 4$. [4]

A salt crystal consist of an array of positive Na and negative Cl ions, both carrying an elementary charge of magnitude e. Assume that a small 'seed' crystal consists of four ions, forming a square of side 0.20nm, as shown in Figure 1.

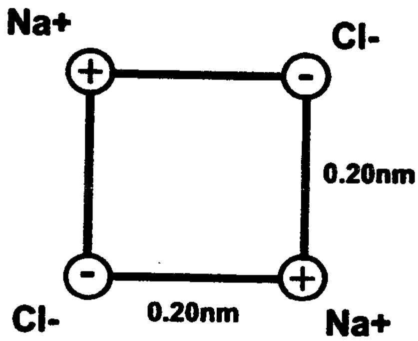

Figure 1:

(a) Calculate the total potential energy of the cluster. [4]

(b) Calculate the work done in removing one of the ions from the cluster, assuming that the other three ions remain fixed. [3]

(c) Find the magnitude and direction of the electric force acting on one of the chlorine ions due to the other ions of the cluster. [7]

(d) Sketch the electric field lines due to the four charges. [2]

8. The Biot-Savart law may be written as

$$\mathrm{d}\underline{B} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\underline{s} \times \hat{\underline{r}}}{r^2}$$

Define the variables used in this expression. [4]

(a) An infinitely long thin wire is bent into the shape shown in Figure 2. Find the magnitude and direction of the magnetic field at the point P in Figure 2, the centre of curvature of the circular segment of the wire, if a current of 10 A flows in the wire. The radius R of the circular segment is 1.25cm. [8]

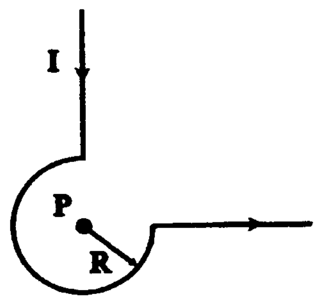

Figure 2:

(b) Use Ampère's law for steady currents to determine the magnetic field at a distance r from an infinitely long straight wire. [3]

An infinitely long wire is bent into the shape shown in Figure 3 and a current I flows in the direction shown. By using Ampère's law or otherwise, calculate the magnitude and direction of the magnetic field at the point P. [5]

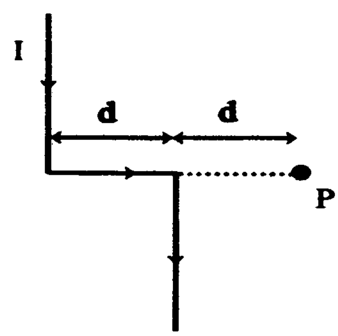

Figure 3:

9. (a) State Faraday's law of electromagnetic induction in words and as an equation. [3]

(b) Describe briefly, and account for, what happens to the current in a closed conducting coil when the north pole of a permanent magnet is first inserted and then withdrawn from the coil. [4]

(c) A conducting bar of length L rotates with constant angular velocity ω about a pivot at one end. A uniform magnetic field $\underline{B}$ is directed perpendicular to the plane of rotation. Show that the motional emf, $\mathcal{E}$, induced between the ends of the bar is

$$\mathcal{E} = \frac{1}{2} B\omega L^2.$$

[8]

(d) A car has a vertical radio aerial 1 metre long. The car is travelling at 60 km/h on a horizontal road in a region where the horizontal component of the Earth's magnetic field is 20 μT towards the north. Calculate the maximum motional emf that can be generated between the ends of the aerial. In which direction should the car travel to generate the maximum emf? [5]

10. (a) State Gauss' law of electrostatics. [2]

(b) A charge Q is distributed uniformly over the surface of a spherical shell of radius a. Use Gauss' law to derive the electric field

i. for $r < a$, and

ii. for $r > a$,

[4]

and make a sketch of the field as a function of distance from the centre of the sphere. [1]

(c) By considering the work done in bringing an infinitesimal charge dQ from infinity up to a spherical shell of total charge Q and radius a, show that the electrostatic energy U of the charged shell is given by

$$U = \frac{Q^2}{8\pi\epsilon_0 a}$$

. [5]

Hence determine the capacitance of a vacuum-filled spherical capacitor consisting of a conducting sphere of radius a. [2]

(d) A hollow conducting sphere is surrounded by a larger concentric, spherical, conducting shell of non-negligible thickness. The inner sphere has a charge $-Q$, and the outer shell a net charge of $3Q$ in electrostatic equilibrium. Use Gauss' law to deduce the charges and electric fields everywhere. [6]

11. (a) Define current density. State its relation to electrical conductivity for an Ohmic material, and hence obtain an expression for the resistance of a wire of length l and uniform cross-sectional area A. [5]

(b) Three copper wires of equal length are connected in series. Their cross-sectional areas are 1.0 mm^2, 2.0 mm^2 and 4.0 mm^2. Determine the voltage drop across the 2.0 mm^2 wire if a voltage of 140 V is applied across them. [5]

(c) Figure 4 shows a circuit containing a charged capacitor and an inductor connected by wires of negligible resistance. After the switch is closed the total energy stored in the circuit is

$$U = \frac{Q^2}{2C} + \frac{1}{2} LI^2,$$

where Q is the charge on the capacitor and I is the current in the circuit. Show that the charge on the capacitor and current in the circuit oscillate sinusoidally and show that the angular frequency of the oscillation is

$$\omega = 1/\sqrt{LC}$$

[6]

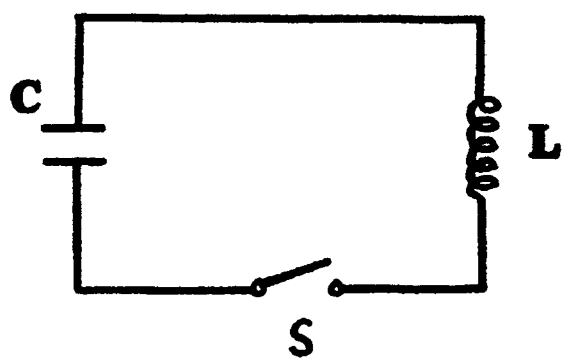

Figure 4:

(d) In the circuit shown in Figure 4, the plates of the capacitor initially carry charges of $\pm Q_0$ and at time $t = 0$ the switch is closed. After a time T the energy stored in the capacitor has fallen monotonically to 3/4 of its intial value. Determine the inductance L in terms of T and C. [4]

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