UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualifications:-

B.Sc. *M.Sci.*

Phys & Astro 2201: Electricity and Magnetism

COURSE CODE	:	**PHAS2201**
UNIT VALUE	:	**0.50**
DATE	:	**16-MAY-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

TURN OVER

Answer *ALL* SIX questions from Section A and THREE questions from Section B.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

permittivity of free space, $\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
electron's charge, $e = 1.602 \times 10^{-19}$ C

SECTION A

[Part marks]

1. (a) Write down the expression for the electric field generated by a point-like charge q. Define the constants appearing in the expression. [2]
   (b) Sketch the electric field lines for a point positive charge. [1]
   (c) By using the principle of superposition, write down the electric field due to a collection of point charges q_1, q_2.... q_n. [2]
   (d) Sketch the electric field lines for two equal point positive charges. [2]

2. (a) Define the flux of the electric field. [1]
   (b) State Gauss' law for electrostatics in integral form. [2]
   (c) By using this law determine the electric field $\underline{E}$ outside of an insulating sphere of radius R uniformly charged with total charge Q. [4]

3. (a) Define a conductor. [2]
   (b) Describe the electric field inside a conductor at electrostatic equilibrium, and explain the answer. [2]
   (c) If an isolated conductor carries a charge, where is the charge distributed? Justify your answer. [2]

4. (a) Write down the Lorenz force, defining the quantities involved. [2]
   (b) Write down Ampère's law, defining the quantities involved. [2]
   (c) By using Ampère's law, derive the magnetic field due to a current in an infinitely long wire. [3]

5. (a) Write down the definition of the flux of the magnetic field, defining the quantities involved. [2]
   (b) Write down Faraday's law. [2]
   (c) Consider a loop of wire, in which a current is induced following a change in the magnetic flux through the loop. Relate the direction of the induced current to the change in the magnetic field flux (Lenz's law). [3]

6. (a) Define the current I through a surface A. [2]

(b) Write down Ohm's law, which relates the current I flowing through a resistor of resistance R to the potential difference V across the resistor. [2]

(c) Write down an expression relating the power dissipated in a resistor of resistance R to the potential difference V across the resistor. [2]

SECTION B

7. (a) Consider an infinite plane sheet, uniformly charged with charge density σ. By using Gauss's law, determine the magnitude of the electric field at a distance z from the plane. [5]

(b) Consider a circular disk of radius R, uniformly charged with charge density σ.

i. Show that the electric field at point P, a distance $z > 0$ from the disk along its central axis, is

$$E(z) = \frac{Qz}{2\pi\epsilon_0 R^2}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]$$

where Q is the total charge on the disk. [5]

ii. What do you expect for $R \to +\infty$? [2]

(c) Consider now a flat ring, with internal diameter R and external diameter $2R$, uniformly charged with charge density σ.

i. Determine the electric field at a point P, a distance $z > 0$ from the ring along its central axis. [4]

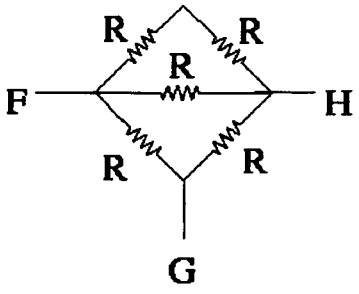
ii. Determine the leading order of the potential V at a point P, located at a large distance $z >> R$ from the ring along its central axis. [4]

8. (a) Write down the equivalent resistance for:

i. Two resistors R_1,R_2 in series; [2]

ii. Two resistors R_1,R_2 in parallel. [2]

(b) Consider the following circuit consisting of 5 equal resistors with resistance R.



i. Determine the equivalent resistance between points F and H. [3]

ii. Determine the equivalent resistance between points F and G. [3]

(c) Define the capacitance for a pair of conductors in terms of the charge stored and the potential between them. [2]

(d) Determine the capacitance of a parallel plate capacitor in terms of the area and separation of the plates, stating all assumptions made. [2]

(e) A capacitor with an initial potential difference of 100 V is discharged through a resistor when a switch between them is closed at $t = 0$. At $t = 10$ s the potential difference across the capacitor is 1 V.

i. What is the time constant of the circuit? [3]

ii. What is the potential difference across the capacitor at $t = 17$ s? [3]

9. (a) Define the force $\underline{F}_B$ acting on a charge q moving with velocity $\underline{v}$ in the presence of a magnetic field $\underline{B}$. [3]

(b) An electron that has velocity

$$\underline{v} = (2.0 \times 10^6)\hat{i} + (3.0 \times 10^6)\hat{j} \text{ m/s}$$

moves through the uniform magnetic field

$$\underline{B} = 0.030\hat{i} - 0.15\hat{j} \text{ T.}$$

Find the force on the electron. [3]

(c) A closed wire loop with current I is in a uniform magnetic field $\underline{B}$.

i. Show that the total magnetic force on the loop is zero. [5]

ii. Does your proof also hold for a nonuniform magnetic field? Justify your answer. [4]

(d) i. Consider a single current loop, of radius R and current I. The magnetic field B at an axial point P a distance z from the centre of the loop is

$$B = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} .$$

Write down an expression for the magnetic field at a large distance from the loop in terms of the loop magnetic moment. [3]

ii. Two concentric, circular wire loops of radius R_1, R_2 are located in the xy plane. They carry a current I_1 and I_2, respectively. Write down an expression for the magnetic field at a point P on the common axis of the two loops, at a large distance from the system of two coils. [2]

10. Consider a circuit consisting of a capacitor, with capacitance C, connected to an inductor, with inductance L. Assume that the inductor has negligible resistance.

(a) Write down an expression for the energy U_E stored in the electric field of the capacitor as a function of C and of the charge Q on the capacitor. [5]

(b) Write down an expression for the energy U_B stored in the magnetic field of the inductor as a function of L and of the current I through the inductor. [5]

(c) Describe the time-dependence of $U_E + U_B$. Justify your answer. [5]

(d) Consider now an RLC circuit consisting of a resistor R, an inductor L and a capacitor C connected in series. Derive a differential equation for the charge q on the capacitor. [5]

11. (a) Define an electric dipole and write down an expression for its electric potential at a large distance. [4]

(b) Define the potential energy U of a system of point charges $q_1, q_2, ..., q_n$. [4]

(c) Given that the potential due to a uniformly charged insulating sphere of radius R and charge Q is

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad (r > R)$$

show that its total electrostatic energy is

$$U = \frac{3}{5}\frac{Q^2}{4\pi\epsilon_0 R}.$$ [6]

(d) Determine the electrostatic energy for a conducting sphere of radius R and total charge Q. [6]

PHAS2201/2006 **END OF PAPER**