## King's College London

## UNIVERSITY OF LONDON

This paper is part of an examination of the College counting towards the award of a degree. Examinations are governed by the College Regulations under the authority of the Academic Board.

M.Sci. EXAMINATION

CP/4750 Image Capture and Sensor Technology

Summer 2001

Time allowed: 3 Hours

Candidates must answer THREE questions. No credit will be given for answering further questions.

The approximate mark for each part of a question is indicated in square brackets.

You must not use your own calculator for this paper. Where necessary, a College calculator will have been supplied.

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## Answer THREE questions

You may use the following values:

Planck's constant	$h = 6.63 \times 10^{-34} \mathrm{J \ s}$
Boltzmann's constant	$k = 1.38 \times 10^{-23} \mathrm{J} \mathrm{K}^{-1}$
Elementary Charge	$e = 1.60 \times 10^{-19} \text{ C}$
Electron charge-to-mass ratio	$e/m = 1.76 \times 10^{11} \ {\rm C \ kg^{-1}}$
Vacuum permittivity	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F m}^{-1}$
Vacuum speed of light	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Energy of $1\mu m$ wavelength photon	$\frac{h\nu}{e} = 1.24 \text{ eV}$
Room temperature	= 300  K
Parallel plate capacitance	$C = \frac{A\epsilon_r \epsilon_0}{d}$ of area A, separation d,
	and relative permittivity $\epsilon_r$ .

[2 marks]

1)	Briefly	explain	each	of the	following	terms:	
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- a) quantum limited detection,[2 marks]b) detector quantum efficiency,
- c) noise equivalent power, [2 marks]
- d) noise power spectral density, and

e) the relationship of Wiener-Khintchine.

[2 marks]

[2 marks]

Draw a labelled diagram showing the key elements of a photomultiplier and explain the principles of its operation.

[5 marks]

What are the main advantages and disadvantages of this type of detector? [3 marks]

Why is it desirable to operate a photomultipler at a voltage where the secondary electron emission is limited to approximately 5 electrons per stage? Calculate the output current for a 12 stage device operated in this manner, when the photocathode receives  $10^{-12}$  W of illumination at a wavelength of 0.5  $\mu$ m. You may assume that the quantum efficiency is 10%.

[3 marks]

Show that the mean squared noise at the output of a photomultiplier may be expressed in terms of the signal limited noise fluctuations  $(\bar{i}_N^2)_{\rm SL}$  and an average secondary electron multiplication factor m by

$$(\overline{i}_N^2)_{\text{out}} = \Gamma G(\overline{i}_N^2)_{\text{SL}}$$

where N is the number of stages, G is the signal power gain and  $\Gamma = \frac{m}{m-1}$  is an excess noise factor over that of signal limited noise fluctuations.

You may assume that Poisson statistics may be applied to the probability of secondary electron emission, and

$$\sum_{n=1}^{N} x^{-(n+1)} \approx \frac{1}{x(x-1)}, \qquad N \gg 1.$$

[9 marks]

SEE NEXT PAGE

2) Describe the principles of operation of the following detector circuit, which utilises a photodiode (P.D.) and an *ideal* operational amplifier (op-amp).

[5 marks]



Explain the significance of each of the following elements of the diagram

- a) the supplied voltage  $-V_{\rm A}$ ,
- b) the supplied current  $I_{\rm D}$ ,
- c) the resistor value  $R_{\rm Tr}$ , and
- d) the capacitance value  $C_{\rm B}$ .

Given that the photodiode is illuminated at an intensity of 10  $\mu$ W cm<sup>-2</sup>, for which it has a spectral sensitivity of 0.4 Amps W<sup>-1</sup> cm<sup>-2</sup>, calculate the voltage developed at  $V_{\text{Out}}$  if  $R_{\text{Tr}} = 1 \text{ M}\Omega$  and the area of the photodiode is 0.5 mm × 0.5 mm.

[3 marks]

Show that the width w of the depletion region at a p-n junction with  $N_{\rm D} \gg N_{\rm A}$ for a photodiode with reverse bias  $-V_{\rm A}$  may be expressed as

$$w = \sqrt{\frac{2\epsilon_r\epsilon_0\left(-V_{\rm A}+\phi\right)}{eN_{\rm A}\left(1+\frac{N_{\rm A}}{N_{\rm D}}\right)}}$$

and explain the meaning of the symbols.

[10 marks]

A p-n junction photodiode has  $N_{\rm A} = 10^{15} \text{ cm}^{-3}$ ,  $N_{\rm D} = 10^{17} \text{ cm}^{-3}$ ,  $\phi = 0.1$  Volts,  $\epsilon_r = 20$  and an area of 0.5 mm× 0.5 mm. Estimate the upper limit for the frequency response when the photodiode is operated at a reverse bias of 2.0 Volts. Comment on an appropriate value for  $C_{\rm B}$ .

[8 marks]

SEE NEXT PAGE

[1 mark] [1 mark]

[1 mark]

[1 mark]

3) Describe the nature of blackbody radiation. State Stefan's Law and Wien's Displacement Law.

[5 marks]

Given that the photon spectral density from a blackbody follows the distribution

$$\rho(\nu) = \frac{\left(\frac{h\nu}{kT}\right)^3}{\exp\left(\frac{h\nu}{kT}\right) - 1},$$

show that, in the limit of low frequency, this distribution may be approximated by  $\rho(\nu) \propto \nu^2$ .

[5 marks]

Calculate the percentage energy in the frequency interval from 30 to 31 GHz, which is emitted from a blackbody at room temperature (300 K).

You may assume that

$$\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}.$$

[5 marks]

Explain the terms *quantum noise* and *Johnson noise* and describe the spectral density of these noise processes. Write down expressions for the mean squared signal current fluctuations caused by:

- a) quantum fluctuations in a signal of average current I, and
- b) arising from Johnson noise in an electrical circuit with resistance R.

[5 marks]

A television set is connected to an antenna which has an impedance of 75  $\Omega$  and a gain of 15 dB. In the frequency range from 300 to 310 MHz, the television displays a noise pattern when it is not tuned to a broadcast station. Estimate what fraction of this noise arises from the existence of the cosmological microwave background radiation (CMBR), which you may assume to be an isotropic source of blackbody radiation with characteristic temperature 3 K.

[Note: there are  $10^9$  CMBR photons per cubic metre of space and the television may be considered as an *ideal* detector apart from the thermal noise present at the antenna, which may be assumed to be at room temperature.]

[10 marks]

4) Describe the spectral characteristics of electromagnetic wave propagation through the atmosphere. Explain the mechanisms which give rise to attenuation and discuss those spectral bands where relatively low losses occur. What is the technological significance of these spectral bands?

[13 marks]

An artificial satellite in Earth-synchronous orbit, 40 000 km above an ocean, is equipped with a 5 Watt laser which has a beam divergence of 1 milliradian. Why is approximately 0.5  $\mu$ m an appropriate wavelength for the operation of the laser so that a communications channel, of moderately high data rate, may be established to a submarine just beneath the ocean's surface?

[2 marks]

A submarine has a circular photodetector array of diameter 1 m, coupled to an ideal photon detector. Calculate the maximum rate at which data can be transmitted from the satellite to the submarine, which will achieve a signal to noise ratio better than 20 dB.

[15 marks]

- 5) Describe, using diagrams where appropriate:
  - a) scanning imaging systems that utilise only single-element detectors. Your answers should refer to key parameters which must be optimised in the system and consider the relative merits of image plane, object plane and afocal scanning techniques, and

[15 marks]

b) primary aberrations in optical lens systems designed for the visible and the infrared regions of the spectrum.

[15 marks]