Queen Mary
University of London

MSci Examination

PHY915 Advanced Topics in Astrophysics

Time allowed: 2 hours 15 minutes

Date:

Time:

Data:	Solar mass	$M_\odot = 2.0 \times 10^{30}$ kg
	Gravitational constant	$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}
	Speed of light,	$c = 3 \times 10^8$ m s^{-1}
	Parsec	1 pc = 3.085×10^{16} m

Answer any eight questions from Section A and any three questions from Section B

SECTION A: Answer any eight questions

All questions carry 6 marks

A1. Explain briefly and simply why the inverse of the Hubble constant is a measure of the age of the universe.

A2. State the cosmological principle. Show that Hubble's law is consistent with the cosmological principle.

A3. Briefly summarise the key physical characteristics of each phase in the evolution of the Universe in chronological order from the big bang to today.

A4. Briefly summarise the observational evidence supporting the standard big bang theory, and mention some observed features of the universe for which the theory does not provide a natural explanation.

A5. Sketch the Lilly-Madau plot of the rest-frame UV luminosity density of distant galaxies versus redhift, as determined form optical and UV observations. Comment on the implications of this plot for the star formation and element production history of the universe.

A6. How does the angular resolution of a telescope depend on the wavelength of observation and the diameter of the primary mirror? Briefly explain the phenomenon of source confusion. How is the confusion-limited sensitivity of a telescope defined? Explain the consequences of confusion for far infrared and submillimetre deep survey observations of high-redshift galaxies

A7. Briefly describe the collapsar model of the origin of long duration gamma-ray bursters (GRBs). What is the baryon loading problem and what type of star is favoured in this model? What is an alternative model of the origin of GRBs ? Explain which model best fits a GRB observed in a dwarf galaxy.

A8. Briefly discuss the significance of non-detections of afterglows fromgamma-ray bursters in the radio and optical wavebands. Mention any improvements that are needed in the current instrumentation.

A9. Explain what is meant by the term reflex motion. Jupiter has a mass of $9.5 \times 10^{-4} M_{\odot}$ and an orbital period of 11.9 years. Use the expression quoted in Question B5:

$$K = \left[\frac{2\pi G}{P}\right]^{1/3} \frac{M_p \sin i}{\left(M_p + M_*\right)^{2/3}},$$

to estimate amplitude of the reflex velocity of the Sun due to Jupiter.

A10. Sketch the mass distribution of the fifty or so extrasolar planets discovered so far. Why is the finding of Jupiter-mass planets in close orbits about solar-type stars surprising in the light of our present understanding of the solar system.

SECTION B: Answer any three questions

All questions carry 50 marks

B1. The Robertson-Walker line-element is in spherical polar co-ordinates (t,r,θ,ϕ) is

$$c^2 dt^2 = a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right],$$

where $a(t)$ is the scale factor of the universe. By considering an incoming radial light ray from a distant galaxy, show the photons emitted at epoch t_e and received at epoch t_r suffer a redshift z given by

$$1 + z := \frac{\lambda_r}{\lambda_e} = \frac{a(t_r)}{a(t_e)}.$$

[20 marks]

The equation of motion for the scale factor is

$$\dot{a}^2(t) = \frac{1}{3}\left[8\pi G\rho(t) + \Lambda\right]a^2(t) - kc^2,$$

where $\rho(t)$ is the density of the universe, Λ is the cosmological constant and $k = \pm 1$ or zero. For each value of k, state the geometry of the universe. **[5 marks]**

Show that the equation for the scale facto can be rewritten in the form

$$-\frac{kc^2}{a^2} = H^2(t)\left[1 - \Omega(t)\right],$$

where

$$\Omega(t) = \Omega_\rho(\tau) + \Omega_\Lambda(t) = \frac{\rho(\tau)}{\rho_c(\tau)} + \frac{\Lambda/8\pi G}{\rho_c(\tau)}; \qquad \rho_c(t) := \frac{3H^2(\tau)}{8\pi G}$$

and where $H(t)$ is the Hubble parameter. **[15 marks]**

Why is $\rho_c(t)$ called the *critical density*? **[3 marks]**

Discuss our current understanding of the values of $\Omega_{\rho o}$ and $\Omega_{\Lambda o}$. **[7 marks]**

B2. Starting from the Friedmann equations,

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi}{3}G\rho + \frac{\Lambda}{3}$$

and,
$$\frac{2\ddot{a}}{a} + \frac{\dot{a}^2 + kc^2}{a^2} = -\frac{8\pi}{c^2}Gp + \Lambda$$

where $a(t)$ is the scale-factor of the universe, G is the gravitational constant, ρ is the density and p the pressure of matter, and a dot denotes differentiation with respect to epoch,how that the deceleration $\ddot{a}$ of the scale factorcan be written in the form

$$\frac{2\ddot{a}}{a} = -\left[\frac{8\pi}{3c^2}G\left(\rho c^2 + 3p\right) - \frac{2\Lambda}{3}\right],$$ **[6 marks]**

Λ being the cosmological constant. Determine the condition for a universe with zero cosmological constant to be open and ever-expanding. What is the implication for the evolution of a universe of Λ's being positive? **[8 marks]**

Show that the scale factor expands exponentially in a ***flat*** de Sitter Universe that has a constant matter density. **[6 marks]**

The particle horizon in a flat, $\Omega = 1$, universe is defined by:

$$\sigma_p = c\int_0^{t_o} \frac{dt'}{a(t')}$$

Show that for a de Sitter universe this tends to a constant value of c/H, where H is the Hubble parameter. **[7 marks]**

What is the implication of this result and why is it different from the particle horizon in a Friedmann universe? **[8 marks]**

Describe the fundamental concepts of inflation theory for the early universe in particular noting how the concepts above help to solve the horizon and flatness problems. Sketch how the scale factor evolves with time from the beginning to today. **[15 marks]**

B3. **(a)** Show that for a survey of a population of uniformly luminous non-evolving galaxies which are randomly distributed in a Euclidean space, the integral source counts, $N(>S)$ is proportional to $S^{-3/2}$, where $N(>S)$ is the number of sources detected per unit solid angle and S is the flux density limit of the survey. **[15 marks]**

Describe briefly and qualitatively the effects on the integral source counts of these assumptions: (i) the galaxy population has a luminosity function which does not evolve; (ii) the luminosity function evolves, with galaxies tending to be brighter at higher redshift. **[10 marks]**

(b) Discuss the main advantages and limitations of far infrared and submillimetre observations in determining the nature of high-redshift star-forming galaxies. Your answer should be as comprehensive as possible, while not necessarily going into detail on any particular aspect. **[25 marks]**

B4. **(a)** With the aid of a sketch, describe the afterglow spectrum from a gamma-ray burster at late times (greater than 1 hour). **[20 marks]**
Describe the appearance of the optical afterglow from a beamed GRB at early and late times, assuming that our line of sight is directly along the jet axis. **[8 marks]**

(b) GRB111111 is observed to have the following properties:

- γ-ray fluence of (6.0 ± 0.3) x 10^{-6} erg cm^{-2}
- a strong [OII] 3727 Å emission line and a weak [Ne III] 3869 Å emission line are detected in a spectrum of the GRB host galaxy. The [OIII] 3727 Å is observed at a wavelength of 7454 Å
- U and B band photometry (corrected for Galactic extinction) given by (these fluxes have been corrected for Galactic extinction)

Optical Filter	Central Wavelength (μm)	Flux density (mJy)	Time since start of GRB (min.)
B	0.55	3.20	200
U	0.365	1.54	250
U	0.365	1.28	300

- at 200 min after the GRB, the cooling frequency $\nu_c = 7.01$ x 10^{13} Hz and the peak emission is at $\nu_m = 5.03$ x 10^{12} Hz.

Assume the following:

1. Luminosity distance $$d_L = \frac{2c(1+z-\sqrt{1+z})}{H_o}$$ where z is the redshift.
2. $H_o = 70$ km s^{-1} Mpc^{-1}
3. $\Omega = 1$
4. $\Lambda = 0$
5. The efficiency of converting shock energy into γ-rays is 0.7 %
6. The electrons are accelerated in the shock with a power law distribution of Lorentz factors $N(\gamma_e) \propto \gamma_e^{-p}$, where $p = 2.5$.

(i) Calculate the distance to the source. **[5 marks]**

(ii) Calculate the isotropic γ-ray energy and the blast wave energy, E_o. **[5 marks]**

(iii) Calculate the spectral index, α, for these observations. Indicate on a your sketch of the afterglow spectrum which portion of the spectrum we are observing. Hint: Assume the temporal decay rate, $F_\upsilon \propto t^\delta$ is constant during these observations and the same rate, δ, applies to both filter bands. **[5 marks]**

Using the values for the spectral index from your sketch of the afterglow spectrum, derive an expression for the peak flux in terms of the observed flux in the B band at 200 minutes after the start of the GRB. Calculate the peak flux at this time. **[7 marks]**

B5. **(a)** A planet of mass M_p orbits a star of mass M_*. Kepler's Third Law relating the period P to the semi-major axis, a, states that

$$P^2 = \left[\frac{4\pi^2}{G}\right]\frac{a^3}{(M_p + M_*)} .$$

Assuming circular orbits, and making use of Kepler's third law, show that the observed line-of-sight velocity amplitude of the star is given by

$$K = \left[\frac{2\pi G}{P}\right]^{1/3} \frac{M_p \sin i}{(M_p + M_*)^{2/3}} ,$$ **[10 marks]**

where i is the inclination of the orbits. How is this expression modified for eccentric orbits**?**

b) Give an account of the reflex method for finding extrasolar planets. Your account should include an explanation of what is measured, an outline of the method currently used in the Lick search programme to attain the highest Doppler precision, why the method is biased towards finding heavy planets in close orbits, a comment about how both the eccentricity and the semi-major axis are obtained, and how evidence is obtained for more than one orbiting planet.
[40 marks]

End of paper