Queen Mary and Westfield College

UNIVERSITY OF LONDON

MSci EXAMINATION

PHY914 (4604): REMOTE SENSING OF ATOMIC AND MOLECULAR SPECIES

Date: 17 May, 2000 Time: 14.30

Time Allowed: TWO HOURS 30 MINUTES

Answer **THREE** questions only. No credit will be given for attempting a further question.

Each question carries 50 marks. The mark *provisionally allocated* to each section is indicated in the margin.

FORMULA:

$$\sum_{n=0}^{\infty} \exp^{-nx} = [1 - \exp^{-x}]^{-1} \qquad \text{where n is integer}$$

DATA:

Intensity of black body radiation (power/unit area/unit solid angle/unit frequency):

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Planck's constant	:	$h = 6.63 \times 10^{-34}$	J s
Speed of light	:	$c = 3.00 \times 10^{8}$	m s^{-1}
Boltzmann's constant	:	$k_B = 1.38 \times 10^{-23}$	J K^{-1}

TURN OVER WHEN INSTRUCTED

1. By considering the energy crossing elements of area at two points along the direction of travel of radiation show that, in empty space, the *intensity* I is constant along this direction. **[8 marks]**

The m^{th} moment $M^{(n)}$ of the specific intensity with respect to a given direction is defined by

$$M^{(n)} := \frac{1}{4\pi} \int_{4\pi} I(\theta) \cos^n \theta \, d\Omega ,$$

where θ is the angle between the direction of $I(\theta)$ and the given direction, and $d\Omega$ is an element of solid angle. Show that the first moment (n=1) is equal to $(4\pi)^{-1}$ times the flux-density F. **[7 marks]**

The equation of radiative transfer governing the rate of change of the intensity $I(\nu)$ with distance s along the direction of travel is

$$\frac{dI}{ds} = j - \kappa I ,$$

where j is the volume emission coefficient and κ is the volume absorption coefficient. Show that the intensity $I(\tau)$ emerging from a cloud of material of optical depth τ is given by

$$I(\tau) = I(0) e^{-\tau} + \int_0^{\tau} S(\tau') e^{-(\tau - \tau')} d\tau' ,$$

where $I(0)$ is the intensity entering the far side of the cloud and $S(\tau)$ is the source function at τ:

$$S(\tau) = \frac{j(\tau)}{\kappa(\tau)} .$$

[15 marks]

Consider radiative transitions between two states i and j, with energies ε_i and ε_j respectively ($\varepsilon_j > \varepsilon_i$), of an ensemble of quantum systems. Let n_i and n_j be the number of systems per unit volume in the two states. Show that, in terms of the Einstein coefficients B_{ji} and B_{ij}, the absorption coefficient $\kappa(\nu)$ at frequency ν in the spectral line produced by transitions between these two levels, is related to the Einstein coefficients by

$$\kappa(\nu) = h\nu_o \left[n_i B_{ij} - n_j B_{ji} \right] \phi(\nu) ,$$

where $\phi(\nu)$ is the line profile, normalised so that,

$$\int_{\text{line}} \phi(\nu) d\nu = 1 ,$$

and $$h\nu_o = \varepsilon_j - \varepsilon_i .$$

[20 marks]

2. The Boltzmann population N_i of energy level E_i of a system of N molecules in equilibrium is given by

$$N_i = \frac{Nd_i}{Q(T)}\exp\left(-\frac{E_i}{kT}\right)$$

where d_i is the degeneracy, T the absolute temperature, k the Boltzmann constant and $Q(T)$ is the partition function given by,

$$Q(T) = \sum_{n=0}^{\infty} d_n \exp\left(-\frac{E_n}{kT}\right).$$

Assuming the vibrational states of a diatomic molecule are all nondegenerate, with vibrational energies approximated by,

$$E(v) = \frac{h}{2\pi}\omega\left(v + \frac{1}{2}\right); \qquad (v = 0,1,2,....),$$

where ω is the angular frequency, show that the number of molecules in state v can be written as:

$$N_v = \frac{N}{Q(T)_{vib}}\exp\left(-\frac{h}{2\pi}\frac{\omega v}{kT}\right)$$

and determine an expression for the vibrational partition function. **[10 marks]**

Assuming that the rotational energies for this heteronuclear, diatomic rigid-rotator molecule are $E(J) = hcBJ(J+1)$, $(J = 0, 1, 2, ...)$, where B is the rotational constant, show that the population of energy level $E_{v,J}$ is give approximately by

$$N_{v,J} = \frac{N(2J+1)}{Q(T)_{vib}Q(T)_{rot}}\exp\left(-\frac{h}{kT}\left(\left(\frac{\omega v}{2\pi}\right) + cBJ(J+1)\right)\right)$$

and that the rotational partion function can be approximated as

$$Q(T)_{rot} \cong \frac{kT}{hcB}$$

and thus determine an expression for the vibration/rotation level population. **[15 marks]**

Find the value of J which maximises the population of a given level $E_{v,J}$. **[10 marks]**

With sketchs illustrate how the expected rotational spectrum changes with increasing temperature. **[10 marks]**

Define the rotational temperature of a gas. **[5 marks]**

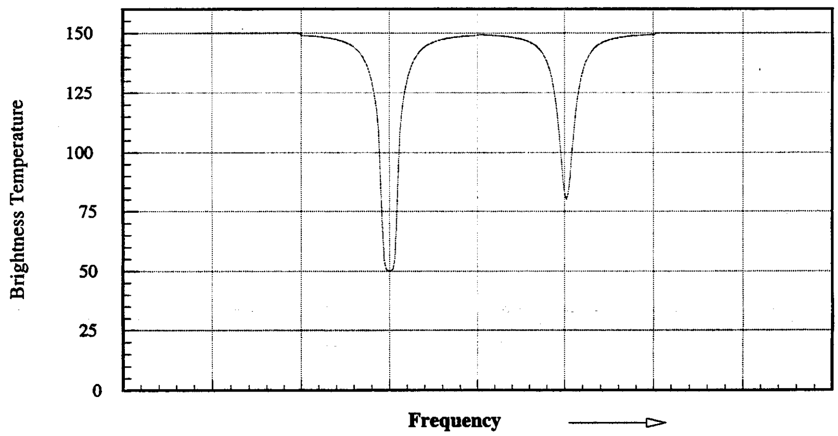
3. The intensity $I(\bar{\nu})$ emitted vertically by a single isothermal atmospheric layer at wavenumber $\bar{\nu}$ is given by:

$$I(\bar{\nu})=B[\bar{\nu},T]\{1-\exp(-\tau(\bar{\nu}))\},$$

where T is the temperature of the layer and $\tau(\bar{\nu})$ is its opacity. Derive an expression for the net intensity received at frequency $\bar{\nu}$ at the top of a multi-layer atmosphere with n layers. Indicate how you could account for emission from a solid planetary surface in your model. **[12 marks]**

The Rayleigh-Jeans approximation can be used for observations made in the far infrared region of the spectrum and the intensity can be expressed in terms of a measured temperature T_m. Re-cast your model in this form. **[8 marks]**

The sketch below shows the absorption from two lines of the same species in an isothermal planetary atmosphere.



The spectral line database shows that one line is twice as strong as the other. Use a single layer model atmosphere to deduce the following:

i. the surface temp

ii. the atmospheric temperature

iii. the opacity of the weaker line **[15 marks]**

Define what is meant by lapse rate. **[5 marks]**

If the above planetary atmosphere were to have had a negative lapse rate, sketch how the stronger line profile might then have looked look. Why are such lapse rate phenomena useful in interpreting planetary atmosphere data? **[10 marks]**

4. Discuss **briefly** the different uses of Nadir and Limb sounding geometries. **[10 marks]**

It can be shown that the intensity emitted vertically by a stratified atmosphere is given by

$$I(\bar{\nu})=\int_0^\infty B[\bar{\nu},T(z)]\kappa(z,\bar{\nu})\exp\left(-\int_0^z \kappa(z',\bar{\nu})dz'\right)dz,$$

where z is the altitude in the atmosphere, $B[\bar{\nu},T(z)]$ is the Planck function at temperature $T(z)$ and $\kappa(z,\bar{\nu})$ is the absorption coefficient at frequency $\bar{\nu}$ and $\alpha(z,\bar{\nu})=\exp\left(-\int_0^z \kappa(z',\bar{\nu})dz'\right)$ is the atmospheric transmittance. Show that the expression for the intensity can be re-written as

$$I(\bar{\nu})=\int_0^\infty B(\bar{\nu},T(z))W(z,\bar{\nu})dz$$

and determine an expression for $W(z,\bar{\nu})$. **[15 marks]**

For a single doppler-broadened spectral line of strength S centered at frequency $\bar{\nu}_o$, the absorption coefficient $\kappa(\bar{\nu})$ is,

$$\kappa(\bar{\nu})=S\frac{2(\ln 2)^{1/2}}{\pi w_D}\exp\left(-\frac{4\ln 2(\bar{\nu}-\bar{\nu}_o)^2}{w_D^2}\right)$$

where w_D is the Doppler line width. For a uniformly-mixed absorber in hydrostatic equilibrium, $dp/dz = -g\rho$, where g is acceleration due to gravity and ρ is the density. Show that

$$\alpha(\bar{\nu})=\exp(-\beta p)$$

and, determine an expression for β. **[10 marks]**

With a change of variable from z to $y = -\ln(p)$, the intensity can be written as:

$$I(\bar{\nu})=\int_0^\infty B[\bar{\nu},T(y)]\frac{d\alpha(y,\bar{\nu})}{dy}dy=\int_0^\infty B[\bar{\nu},T(y)]W(y,\bar{\nu})dy$$

Determine the form of $W(y)$ and sketch it as a function of p/p_o where p_o is the pressure where $W(y)$ is a maximum.

The equivalent expression for a pressure broadened line is:

$$W(y)=2\left(\frac{p}{p_o}\right)^2\left\{\exp-\left(\frac{p}{p_o}\right)^2\right\}$$

Overlay this function on your plot and discuss the implications of these results. **[15 Marks]**

Please turn to next page

5. Discuss two modeling approaches leading to the extraction of atmospheric parameters from spectral measurements. **[10 marks]**

Outline the problems associated with extracting atmospheric parameters from the measured spectral emission. **[15 marks]**

Outline two inversion methods that are used to give approximate results. **[25 marks]**

