Queen Mary and Westfield College

UNIVERSITY OF LONDON

MSci EXAMINATION

PHY914: REMOTE SENSING OF ATOMIC AND MOLECULAR SPECIES

Date: 20 May 1999 Time: 10:00

Time Allowed: TWO HOURS 30 MINUTES

Answer **THREE** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

DATA:

Intensity of black body radiation (power/unit area/unit solid angle/unit frequency):

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{(\exp^{h\nu/k_BT} - 1)}$$

Planck's constant	:	$h = 6.63 \times 10^{-34}$	J s
Speed of light	:	$c = 3.00 \times 10^{8}$	m s^{-1}
Boltzmann's constant	:	$k_B = 1.38 \times 10^{-23}$	J K^{-1}

TURN OVER WHEN INSTRUCTED

1. Define the *specific intensity* I_ν of radiation as it appears in problems of radiative transfer. [3 marks]

The solar luminosity is 3.82×10^{26} W and the diameter of the sun subtends an angle of 30 arcsec at the earth. The earth is 150×10^{6} km from the sun. Calculate the *total* intensity of solar radiation above the Earth's atmosphere. [4 marks]

By considering the radiation entering and leaving a small volume of space, show that the propagation of the radiation is governed by the *equation of radiative transfer*:

$$\frac{dI(\nu)}{ds} = j(\nu) - \kappa(\nu)I(\nu)$$

where s is the distance in the direction of travel, $j(\nu)$ is the emission coefficient and $\kappa(\nu)$ is the absorption coefficient. [4 marks]

Consider radiative transitions between two states i and j, with energies ε_i and ε_j respectively ($\varepsilon_j > \varepsilon_i$), of an ensemble of quantum systems. Let n_i and n_j be the number of systems per unit volume in the two states. In terms of the Einstein coefficients A_{ji}, B_{ji} and B_{ij}, the equation of transfer is given, in terms of the Einstein coefficients, by

$$\frac{dI(\nu)}{ds} = h\nu\left[n_j A_{ji} - \left(n_i B_{ij} - n_j B_{ji}\right)I(\nu)\right]\phi(\nu)$$

where $\phi(\nu)$ is the line profile, normalised so that

$$\int_{\text{line}} \phi(\nu)d\nu = 1$$

and $$h\nu = \varepsilon_j - \varepsilon_i .$$

Use a thermodynamic argument to show quite generally that the Einstein coefficients are related by

$$B_{ji} = B_{ij}$$

and $$A_{ji} = \frac{2h\nu^3}{c^2} B_{ji} .$$ [5 marks]

The absorption coefficient $\kappa(\nu)$ can be written as

$$\kappa(\nu) = \kappa_o W \phi(\nu)$$

where $\kappa_o = \kappa(\nu_o)$, ν_o being the frequency at the line centre. Show that

$$\int_{\text{line}} \kappa(\nu)d\nu = \kappa_o W$$ [2 marks]

and use a diagram to give a geometrical interpretation of W. [2 marks]

2. Explain why all spectral lines have a natural width and are not truly monochromatic. What is the function that best describes the line profile? [3 marks]

The natural line shape for spectral emission or absorption is given by the following expression:

$$g_o(\nu) = \frac{\dfrac{1}{\pi\tau_{12}}}{(\nu - \nu_{12})^2 + \dfrac{1}{4(\pi\tau_{12})^2}}$$

where ν_{12} is the frequency of the transition and τ_{12} is the state lifetime. Determine the function maximum and show that the full (frequency) width at half maximum (the natural line width) is given by:

$$\Delta\nu_n = \frac{1}{\pi\tau_{12}}$$

Thence determine the form of the line-shape function in terms of frequency and line width. [6 marks]

Empirically, at a given temperature and at pressure P the pressure-broadened line width is given by $\Delta\nu_p = \gamma P$, where γ is the collision broadening coefficient. Use the above results to write down the expression for the line profile for pressure broadened emission features. [3 marks]

The Doppler line width, $\Delta\nu_D$, the full width at half-maximum intensity of the Gaussian profile, is given by,

$$\Delta\nu_D = 7.16x10^{-7}\left(\frac{T}{M}\right)^{1/2}\bar{\nu}$$

where T is the gas temperature and M its molecular weight. Why is Doppler broadening referred to as inhomogeneous and what causes the line shape to be Gaussian? [3 marks]

A gas, of molecular weight 28, is in an isothermal planetary atmosphere at temperature 250 K and has a pressure broadening coefficient of, $\gamma = 3$ GHz. At a frequency of 1000 cm^{-1} at what pressure does the Doppler broadening equal the pressure broadening. What is the advantage of making remote sensing observations at lower frequencies? [3 Marks]

Sketch the line width as a function of pressure, and therefore height, in this atmosphere for different observation frequencies. [2 Marks]

3. The vibrational energy levels $E(n)$ for an ideal harmonic oscillator are given by

$$E(n) = \frac{h}{2\pi}\left(\frac{k}{\mu}\right)^{1/2}\left(n + \frac{1}{2}\right) = \frac{h}{2\pi}\omega_{osc}\left(n + \frac{1}{2}\right),$$

where ω_{osc} is the angular frequency of the oscillator, k is the force constant, μ is the reduced mass, h is Planck's constant and n is the vibrational quantum number (n=0, 1, 2, ...).

Express the energy in terms of wavenumber. Explain why the expression gives at best an approximation to the actual energy levels of a vibrating diatomic molecule. Use a sketch to illustrate where the model departs from reality. [6 marks]

The selection rule for transitions between the vibrational states of the ideal oscillator only allow $\Delta n = \pm 1$. Sketch the emission spectrum that you expect to observe for both this ideal situation and a real a diatomic molecule. [4 marks]

The rotational energy levels of the diatomic molecule are given by:

$$E = \frac{h^2}{8\pi^2 \mu r_e^2} J(J+1) \qquad (J = 0, 1, 2, ...)$$

How does the inclusion of these energy levels modify the observed spectral characteristics? Your answer should include a staement of the Born-Openheimer approximation, calculation of the new energy levels, the selection rules and a determination of the expected spectrum. [10 marks]

4. The intensity emitted vertically by a single isothermal atmospheric layer at frequency $\overline{\nu}$ is given by:

$$I(\overline{\nu}) = B[\overline{\nu}, T_A]\{1 - \exp(-\tau(\overline{\nu}))\},$$

where T_A is the atmospheric temperature and $\tau(\overline{\nu})$ is its opacity. Derive an expression for the net intensity received at frequency $\overline{\nu}$ at the top of a multi-layer atmosphere with n layers. Indicate how you could account for emission from a solid planetary surface in your model. [7 marks]

Sketch the emission spectrum that you would observe from a uniform atmosphere containing a single Lorenzian line centred near 400 cm^{-1} (25 μm) for the following cases:

i. A surface temperature of 150 K and an atmospheric temperature of 90 K.

ii. A surface temperature of 90 K and an atmospheric temperature of 150 K.

You should indicate the effect of varying line strength in your sketches. [6 marks]

With the aid of a diagram, discuss the temperature structure of the Earth's atmosphere, stating the physical and chemical processes responsible for the various layers. [7 marks]

5. Discuss briefly the different uses of Nadir and Limb sounding geometries. [4 marks]

Show that the transmission α, of the atmosphere along a slant path from a satellite to a point x is given by:

$$\exp\left(-\int_0^x \kappa(x', \overline{\nu})\, dx'\right)$$

where $\kappa(x', \overline{\nu})$ is the absorption coefficient of the Earth's atmosphere at wavenumber $\overline{\nu}$ and slant distance x'. [4 marks]

Use Kirchoff's law to show that the total intensity of radiation received at a limb viewing instrument from all atmospheric layers can be written as:

$$I(\overline{\nu},x) = \int B(\overline{\nu},T(x))\kappa(x,\overline{\nu})dx \exp\left(-\int_0^x \kappa(x',\overline{\nu})dx'\right)$$

where $B(\overline{\nu},T(x))$ is the Planck function for temperature $T(x)$ at slant distance x. [3 marks]

Using the results from above simplify this equation to the form:

$$I(\overline{\nu},x) = \int B(\overline{\nu},T(x))W(x,\overline{\nu})dx$$

explaining briefly what the function $W(z,\overline{\nu})$ represents and why it is important. [4 marks]

The absorber amount in the limb path, x, for a uniformly mixed gas, is:

$$m = \int c\rho(x)\,dx$$

where c is the mass mixing ratio and $\rho(x)$ is the air density. By relating x to z and the tangent height, h show that this can be approximated by:

$$m = 2\int c\rho(z)\sqrt{\frac{R}{2(z-h)}}dz$$

Thus estimate by what factor an absorber emissivity would apparently be increased by if we were to make limb sounding observations from a balloon at 40 km viewing a tangent height of 20 km compared to nadir observations from the same platform. Assume that the radius of the Earth = 6000 km. [5 marks]