Royal Holloway

UNIVERSITY OF LONDON

MSci EXAMINATION

NUCLEAR PHYSICS CP4511A

SUMMER 1999

Time Allowed: TWO AND A HALF HOURS

Answer **THREE** questions only. No credit will be given for attempting any further questions.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

TURN OVER WHEN INSTRUCTED

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	m_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	e_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi e_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$h = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	s	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^3
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

e = 2.718 $\pi = 3.142$ $\log_e 10 = 2.303$

PART MARKS

1. (a) What approximations are made in the Schrödinger equation

$$\frac{\mathrm{d}^2U(r)}{\mathrm{d}r^2}+\frac{M}{\hbar^2}[E-V(r)]U(r)=0\,,$$

in order that it should apply to the case of the deuteron. **[4]**

Using the Hulthén potential, $V_H(r)$, the ground state wave function of the deuteron is found to be:

$$U(r)=(1-\mathrm{e}^{-ar})\mathrm{e}^{-kr}\,,$$

where α and k are constants. Find $V_{\mathrm{H}}(r)$ in terms of α, k and M. **[6]**

$\left[\text{You should consider } V_{\mathrm{H}}\,(r) \text{ in the limit as } r \to \infty \text{ to eliminate } E.\right]$

(b) The deuteron is the lightest complex nucleus that can be formed in a two-body reaction and can also initiate simple nuclear reactions of this type.

(i) Give an example of such a reaction and explain how it can be used to determine experimentally the binding energy of the deuteron. **[5]**

(ii) Calculate the energy of protons detected at 90° when 2.1 MeV deuterons are incident on ^{27}Al to produce ^{28}Al with an energy difference Q = 5.5 MeV. **[5]**

$$\left[\begin{array}{l}\text{In a two-body reaction } x+X\to y+Y, \text{ to a good approximation,}\\ Q=K_y\left(1+\dfrac{A_y}{A_Y}\right)-K_x\left(1-\dfrac{A_x}{A_Y}\right)-2\dfrac{\sqrt{A_xA_y}}{A_Y}\sqrt{K_xK_y}\cos\theta\\ \text{where } K_x \text{ is the energy of incident particles } x \text{ and } K_y \text{ the kinetic energy of}\\ \text{particles } y \text{ scattered through an angle } \theta\text{. The } A \text{ variables are mass numbers.}\end{array}\right]$$

2. Justify the use of a spherically symmetric harmonic oscillator potential as a basis for the nuclear shell model. Without giving a detailed mathematical solution of the Schrödinger equation deduce the energy eigenvalues in terms of the total oscillator quantum number N. **[4]**

Give a diagram for the energy levels up to N=6 showing the allowed orbital angular momentum values and the occupation number for each level. What advantages may be achieved by altering the shape of the potential, and in what way is that not sufficient to provide a good description of level ordering? **[12]**

The spins and parities of the ground and four excited states of $^{207}_{82}\mathrm{Pb}$ are : $\mathrm{I}(\pi)$, $\frac{1}{2}(-)$, $\frac{5}{2}(-)$, $\frac{3}{2}(-)$, $\frac{13}{2}(+)$, $\frac{7}{2}(-)$. Comment on the shell model description of theses states. **[4]**

3. The empirical mass formula (neglecting a term representing the odd-even effect) is

$$M(A,Z) = Z\left(m_p + m_e\right) + (A - Z)m_n - aA + bA^{2/3} + g\,(A - 2Z)^2 / A + eZ^2 A^{-1/3}$$

where α, β, γ and ε are constants.

By finding the minimum in $\mathrm{M}(A,Z)$ for constant A obtain the expression

$$Z_{\min} \cong 0.5A\left(1 + 0.25A^{2/3}\varepsilon/\gamma\right)^{-1}$$

for the value of Z which corresponds to the most stable nucleus for a set of isobars of mass number A. **[10]**

Deduce that with $\varepsilon = 0.72$ MeV and $\gamma = 23$ MeV the ratio $Z_{\min}/A$ is approximately 0.5 for light nuclei and 0.4 for heavy nuclei. **[4]**

State the selection rules governing g -ray transitions between states with spin-parity I_i^p and I_f^p. In the ^{180} Hf nucleus the ground and first five excited states have respectively the following values of I^p : $0^+, 2^+, 4^+, 6^+, 8^+, 8^-$. Use the selection rules to identify the multipolarities of the following g transitions:

$$8^- \to 8^+,\ 8^+ \to 6^+, 6^+ \to 4^+,\ 4^+ \to 2^+,\ 2^+ \to 0^+,\ 8^- \to 6^+.$$

[6]

4. Nuclear reactions can be classified as:

   (i) knockout and transfer reactions,

   (ii) direct and compound nucleus reactions.

Explain the meaning of each classification. **[4]**

(a) The deuteron stripping reactions (d,n) or (d,p) can be considered to take place at the surface of the target nucleus in a direct reaction process. If the incident deuteron has momentum $\mathbf{p_1}$ and the outgoing nucleon momentum $\mathbf{p_2}$ show that the angular momentum l of the orbit into which the transferred nucleon is placed is

$$l^2 = \frac{4c^2 p_1 p_2 \sin^2\left(\theta/2\right)}{\hbar^2 c^2 / R^2}$$

where R is the radius of the target nucleus and θ the angle of emergence of the outgoing nucleon. **[4]**

Single neutron shell – model states in $^{65}\mathrm{Zn}$ are produced by a (d,p) reaction on $^{64}\mathrm{Zn}$ with incident deuterons of 5 MeV (corresponding to $p_d \cong p_p \cong 140$ MeV/c). Show that $l \cong \sqrt{32}\,\sin\left(\theta/2\right)$ and find the angles of emergence of the protons corresponding to angular momentum transference of l = 0, 1 and 2. **[4]**

$$\left[\text{take } \hbar c = 197 \text{ MeV fm}\right].$$

(b) Describe the main characteristics of a compound nucleus reaction. Give four different ways that the compound nucleus resulting from protons bombarding a $^{63}\mathrm{Cu}$ target can decay. What can this model predict for the interaction of alpha particles with a $^{60}\mathrm{Ni}$ target? **[8]**

5. Describe with the aid of a diagram the main features and principles of operation of the Separator for Heavy Ion Products (SHIP) which has been used at Darmstadt to investigate the formation of new elements with Z up to 110. **[6]**

Explain how an element with Z = 109 may be

   (i) produced, **[4]**

   (ii) identified, **[8]**

in a heavy ion reaction.

END

PART MARKS

Briefly explain why the production of new elements by fusion will not occur in the reaction $^{238}_{92}\mathrm{U}+^{208}_{82}\mathrm{Pb}$. **[2]**