UNIVERSITY OF LONDON

MSci EXAMINATION 2000

For Internal Students of

Royal Holloway

DO NOT TURN OVER UNTIL TOLD TO BEGIN

PH4502A: LOW TEMPERATURE PHYSICS AND NANOTECHNOLOGY

Time Allowed: TWO AND A HALF hours

Answer THREE QUESTIONS only

No credit will be given for attempting any further questions

Approximate part-marks for questions are given in the right-hand margin

Calculators ARE permitted

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	m_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	e_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi e_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\hbar = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	s	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^{2} kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^{3}
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

$e \cong 2.718$ $\pi \cong 3.142$ $\log_e 10 \cong 2.303$

PART MARKS

1. ***Either***

Superfluid ^{4}He is described by a macroscopic wavefunction

$$\psi(r) = \psi_0 \exp(\mathrm{i}\,\varphi(r)).$$

(a) Discuss the physical interpretation of this wavefunction in terms of the superfluid density and superfluid velocity, and the phenomenon of Bose-Einstein condensation in this and other systems. **[5]**

(b) Describe briefly how the superfluid density varies with temperature, and the contributions to the normal density from the elementary excitations. **[5]**

(c) Show how the existence of this macroscopic wavefunction leads to quantization of the circulation in a toroidal flow channel, and quantized vortex lines in a multiply connected geometry. Derive the superfluid velocity as a function of distance from the vortex core. **[5]**

(d) A torus of liquid helium is cooled through the superfluid transition temperature while rotating, and the rotation is then stopped. Briefly discuss what determines the angular momentum of the rotating fluid, and how it varies when the temperature is subsequently changed. **[2]**

(e) Discuss the mechanisms which determine the critical velocity for the breakdown of superfluidity. **[3]**

Or

(a) Describe the principle of the Scanning Tunnelling Microscope (STM) in the Constant Current Mode (CCI) and in the Constant Height Mode (CHI). What are the practical limitations of CCI which can be overcome using CHI? **[5]**

(b) Describe the Independent Electrode Approximation (Tersoff-Hamann) model of STM imaging. **[6]**

(c) What is the ‘reciprocity principle’ in STM? **[4]**

(d) Explain the physics of the Coulomb blockade in tunnelling. **[5]**

PART MARKS

2. (a) Describe the phase diagram of isotopic liquid helium mixtures. **[6]**

(b) Account for the limiting finite solubility of ^{3}He in ^{4}He. **[2]**

(c) Give an account of the operation of a ^{3}He-^{4}He dilution refrigerator, which includes an explanation of:

(i) the functions of the mixing chamber and of the still

(ii) the factors determining the ^{3}He concentration in the mixing chamber and in the still.

Briefly discuss the advantages of the dilution refrigerator compared to a ^{3}He refrigerator. **[6]**

(d) The mixing chamber of a dilution refrigerator of minimum temperature 3 mK, operating at a circulation rate of 200 μmole s^{-1}, is subjected to a heat load of 1μW. Calculate the equilibrium temperature under these conditions. **[2]**

(e) This refrigerator is now used to cool a sample with heat capacity $10^{-3}/T^2$ J K^{-1}, from 100 mK to 20 mK. Estimate how long this takes. Also estimate how much longer it will take to cool to 10 mK. **[4]**

[The cooling power of a dilution refrigerator is given by $\dot{\varphi} = 84\,\dot{n}\left(T^2 - T_0^2\right)$, where $\dot{n}$ is the ^{3}He circulation rate (mole s^{-1}) and T_0 is the minimum temperature.]

PART MARKS

3. (a) Discuss fully the assumptions of the two fluid model of superfluid ^4He. Illustrate the model by two experiments that can be understood in terms of it. **[7]**

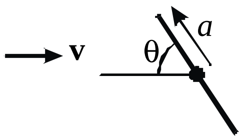
(b) From the data given below calculate the fountain pressure at 1.8 K and at 1.2 K for a temperature difference of 1 mK. **[3]**

(c) Describe briefly the phenomenon of thermal counterflow in superfluid ^4He. Explain why the normal fluid velocity is given by the expression

$$\dot{q} = \frac{\dot{Q}}{A} = T r s v_{\mathrm{n}}$$

where $\dot{q} = \dot{Q}/A$ is the applied heat flux through an area A, and v_{n} is the resulting normal fluid velocity. r, s and T are the total density, entropy per unit mass and temperature respectively. From the data given below calculate the normal and superfluid velocities at 1.2, 1.8 and 2.1 K, when the applied heat flux is 20 W m^{-2}. **[4]**

(d) In classical hydrodynamics it was shown by Lord Rayleigh that, when an inviscid fluid (i.e. a fluid assumed to have zero viscosity) flows past a circular disc, which is mounted on a torsion fibre normal to the fluid flow



and in the plane of the disc, the disc experiences a torque tending to align it perpendicular to the flow given by

$$t = \frac{4}{3} a^3 r v^2 \sin 2q$$

where q is the angle between the flow velocity and the plane of the disc, a is the radius of the disc and r is the density of the fluid.

Show that in superfluid ^4He in the presence of thermal counterflow, treating the normal component as inviscid, the torque on the disc when $\theta = 45°$ is given by

$$t = \frac{4}{3} a^3 \frac{\dot{q}^2}{T^2 s^2 r} \frac{r_{\mathrm{n}}}{r_{\mathrm{s}}}.$$ **[4]**

(e) Sketch the temperature dependence of this torque, and discuss briefly how the disc would respond to a second sound standing wave. **[2]**

Temperature (K)	Entropy (J kg^{-1} K^{-1})	Density (kg m^{-3})	Normal fraction (r_{n}/r)
1.2	52	145	0.029
1.8	535	145	0.32
2.1	1240	145	0.74

PART MARKS

4. (a) Calculate the temperature interval in which a conductor with the parameters outlined below behaves as one-dimensional (1D) with respect to the weak localisation effect. **[7]**

The conductor has the shape of a strip with dimensions $L_x = 20$ μm in the direction of the electric current, $L_y = 300$ nm and $L_z = 10$ nm in the transverse directions; elastic scattering rate for the conduction electrons is 10^{13} s^{-1}; the phase-breaking rate t_ϕ^{-1} changes with temperature according to the law $t_\phi^{-1} = AT^3$ s^{-1}; $A = 10^8$ s^{-1} K^{-3}; spin-orbit scattering is negligible; Fermi velocity $v_F = 10^6$ m s^{-1}.

(b) Calculate the temperature dependence of the weak localisation correction in the 1D case for the conductor with the parameters given above. **[7]**

(c) Describe qualitatively the changes in resistance of the conductor with magnetic field. What is the difference in the behaviour of the conductor in the case of a magnetic field **H**, (i) parallel to the y-axis, and (ii) the x-axis? Analyse the 1D case. **[6]**

5. (a) Describe the positive process for e-beam nanofabrication. **[3]**

(b) Give three examples of nanotechnology techniques based on the use of a scanning probe. **[3]**

(c) Describe the ‘self-alignment’ nanofabrication technique. Give an example used for the fabrication of sub-micron tunnel junctions. **[4]**

(d) Calculate the dependence on contrast of the resolution of a positive e-beam resist using the two-gaussian model for the proximity effect and the isotropic local model for the development. **[7]**

(e) Explain the physical limitations of the resolution of photo-lithography and X-ray lithography. **[3]**