Royal Holloway

UNIVERSITY OF LONDON

MSci EXAMINATION

QUANTUM FLUIDS CP4501A

SUMMER 1999

Time Allowed: TWO HOURS

Answer **TWO** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

TURN OVER WHEN INSTRUCTED

GENERAL PHYSICAL CONSTANTS

Permeability of vacuum	m_0	=	$4\pi \times 10^{-7}$	H m^{-1}
Permittivity of vacuum	e_0	=	8.85×10^{-12}	F m^{-1}
	$1/4\pi e_0$	=	9.0×10^{9}	m F^{-1}
Speed of light in vacuum	c	=	3.00×10^{8}	m s^{-1}
Elementary charge	e	=	1.60×10^{-19}	C
Electron (rest) mass	m_e	=	9.11×10^{-31}	kg
Unified atomic mass constant	m_u	=	1.66×10^{-27}	kg
Proton rest mass	m_p	=	1.67×10^{-27}	kg
Neutron rest mass	m_n	=	1.67×10^{-27}	kg
Ratio of electronic charge to mass	e/m_e	=	1.76×10^{11}	C kg^{-1}
Planck constant	h	=	6.63×10^{-34}	J s
	$\mathrm{h} = h/2\pi$	=	1.05×10^{-34}	J s
Boltzmann constant	k	=	1.38×10^{-23}	J K^{-1}
Stefan-Boltzmann constant	s	=	5.67×10^{-8}	W m^{-2} K^{-4}
Gas constant	R	=	8.31	J mol^{-1} K^{-1}
Avogadro constant	N_A	=	6.02×10^{23}	mol^{-1}
Gravitational constant	G	=	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration due to gravity	g	=	9.81	m s^{-2}
Volume of one mole of an ideal gas at STP		=	2.24×10^{-2}	m^3
One standard atmosphere	P_0	=	1.01×10^{5}	N m^{-2}

MATHEMATICAL CONSTANTS

e = 2.718 $\pi = 3.142$ $\log_e 10 = 2.303$

PART MARKS

1. (a) Sketch the spectrum of elementary excitations in superfluid ^{4}He. Discuss briefly its important features, and how it differs from that of an ideal Bose gas. **[6]**

   (b) Give a full derivation of the Landau critical velocity of superfluid ^{4}He. Compare this result with that for an ideal Bose gas. **[10]**

   (c) Discuss how, in most cases of practical importance, the onset of dissipation in superflow is limited by phase slippage due to the motion of quantized vortex rings, rather than the mechanism discussed by Landau. **[4]**

2. (a) Describe fully the phase diagram of liquid isotopic solutions ^{3}He and ^{4}He. **[6]**

(b) Explain the functions of the *still* and *mixing chamber* of a dilution refrigerator, including a description of the mechanism by which circulation of ^{3}He is achieved and the factors influencing the choice of optimal still temperature. **[5]**

(c) With the information given below estimate the ^{3}He concentration in the still, when operated at 0.7 K. **[2]**

(d) Calculate the molar entropy of pure ^{3}He and a saturated solution of ^{3}He in ^{4}He and hence show that the cooling power at the mixing chamber is given by

$$\dot{Q} = 84\dot{n}T^2 \quad (\mathrm{JK^{-2}mol^{-1}})$$

where $\dot{n}$ is the ^{3}He circulation rate (expressed in mole s^{-1}). **[4]**

(e) An experimentalist wishes to detect gravitational waves, using as an antenna a 100 tonnes sphere of copper cooled by to 10 mK by a dilution refrigerator. Evaluate the claim that the time taken to cool the sphere from 1 K to 10 mK is less than one day. The dilution refrigerator has a circulation rate of 10^{-3} mole s^{-1}. Assume that the molar heat capacity of copper is given by $c = g\,T$, where g = 0.7 mJ K^{-2}mole^{-1}, arising solely from the electrons. [1 tonne = 10^3 kg. The mass of one mole of copper is 0.0635 kg.] **[3]**

[The heat capacity of a Fermi gas is given by

$$c = Nk_B\frac{\pi^2}{2}\frac{T}{T_F} \qquad \text{where} \qquad T_F = \frac{\hbar^2}{2m^*k_B}\left[3\pi^2 N/V\right]^{2/3}$$

is the Fermi temperature, N is the number of fermions in volume V, of effective mass m^*. For pure ^{3}He at saturated vapour pressure, the molar volume is 36.8 cm^3 and the quasiparticle effective mass ratio m^*/m_3 = 2.8. For a saturated solution, the molar volume is 27.5 cm^3 and the effective mass ratio m^*/m_3 = 2.5. The ^{3}He atomic mass m_3= 5.10^{-27} kg.

The osmotic pressure π of a Fermi gas is given by

$$\Pi = \frac{Nk_BT}{V} \quad \text{for} \quad T \geq T_F \quad ; \quad \Pi = \frac{2Nk_BT_F}{5V} \quad \text{for} \quad T << T_F\,. \qquad]$$

PART MARKS

3. (a) Describe the main features of the phase diagram of ^{3}He. Your account should include brief comments on the nature of the various phases, and an explanation of the temperature dependence of the melting curve. **[8]**

(b) Discuss the different Cooper pair wavefunctions in the A and B phases of superfluid ^{3}He and contrast with a conventional metallic superconductor such as lead, tin or zinc. Give examples of how the different pairing state gives rise to distinct behaviour of *two* properties in the superfluid state. **[8]**

(c) A sample of superfluid ^{3}He-A is confined within a slab geometry, i.e. between two flat parallel plates of separation D. Discuss the superfluid textures of $\underline{\boldsymbol{l}}$ and $\underline{\boldsymbol{d}}$ that occur as a function of the magnetic field applied normal to the plates, for D = 1 mm and D = 5 μm. Briefly explain how NMR measurements can be used to characterize these textures. **[4]**

END