Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Permittivity of Free Space: $\varepsilon_o = 8.8 \times 10^{-12}$ F m^{-1}

Electronic Charge: $e = 1.6 \times 10^{-19}$ C

Permeability of Free Space: $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}

Proton Rest Mass: $m_p = 1.67 \times 10^{-27}$ kg

Question 1.

What characterises a plasma? [1]

A laser produces a plasma with an ion number density of 10^{16} m^{-3} in a confined spherical shell of radius 0.005 m. If the confining shell allows 1% of the negative charge to escape from the plasma, determine the electric field at its surface. You may neglect the dielectric properties of the shell. [5]

Consider a cold plasma:

(i) Explain how a local electric field can develop. [2]

(ii) Derive an expression for the electron plasma frequency , ω_{pe}, assuming the electron number density is n_o. Indicate any approximations used. [5]

(iii) How is ω_{pe} related to the ion plasma frequency ω_{pi}. [2]

(iv) Show that $\left|\frac{\omega_{pe}^2}{\omega_{ce}}\right| = \left|\frac{\omega^2{}_{pi}}{\omega_{ci}}\right|$

where ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies. [5]

Question 2

What is the Larmor radius? [1]

Derive an expression for the magnetic moment of an electron moving with a velocity, V, in a B field. [3]

Consider a static inhomogeneous magnetic field which is symmetric about the z-axis, and weakest in the xy plane.

(i) Show that particle trapping will occur when

$$\frac{V_{zo}^2}{V_{\perp o}^2} \leq \left(\frac{B_m - B_o}{B_o} \right)$$

where the subscript 'o' denotes the parameters in the z=0 plane and B_m is the trapping field. [10]

(ii) Sketch the motion of an electron in this field. [2]

Explain how the trapping effect is used in a magnetic bottle spectrometer to spatially resolve electrons with different energies produced by photo-ionisation. [4]

Question 3

Explain what is meant by magnetic flux freezing. [1]

For a conducting fluid carrying a current J in a B field, give the force equation of fluid motion in terms of J, B and hydrostatic pressure, P. [2]

Now show that the force acting on the plasma fluid is

$$-\nabla P - \nabla_{\perp}\left(\frac{B^2}{2\mu_o}\right) + \frac{1}{R}\left(\frac{B^2}{\mu_o}\right)\hat{n}$$

where $\nabla_{\perp}$ acts perpendicularly to the local B field along $\hat{n}$ and R is the radius of curvature of the B field. [8]

Describe the influence of the *pressure* and *tension* of the B field on the plasma fluid. [4]

The solar wind is deflected by the Earth's magnetic field producing a magnetopause in the equatorial plane. Calculate at what distance from the Earth the magnetopause occurs, given that the solar wind has a velocity of 10^5 m s^{-1} with a proton number density of 10^5 m^{-3}. The Earth's magnetic field is given by $3\times10^{-5}R^{-3}$Tesla where R is in Earth radii. [5]

Question 4

Using the Keldysh parameter, explain the difference between *tunnel ionisation* and *multi photon ionisation* occurring in a laser of angular frequency ω. [4]

Consider an electron of velocity V moving through an atomic gas in the presence of a laser field of amplitude E and frequency ω.

(i) Discuss the dynamics of the scattering in terms of the collision time, τ, the time between collisions τ_c and the energy of oscillation. [3]

(ii) By considering the momentum change, show that the equation of motion of the electron is:

$$m\dot{V} = -mV\upsilon_m - eE$$

where υ_m is the collision frequency associated with the momentum charge. [7]

(iii) Taking the solution for V to be:

$$V = \frac{-ieE}{m(\omega + i\upsilon_m)}$$

show that the energy absorbed from the radiation field at each collision is

$$\frac{e^2 E_o^2}{2m\,\omega^2}\left(\frac{\omega^2}{\omega^2 + \upsilon_m^2}\right)$$

where E_o is the maximum amplitude of E. [6]

Question 5

The matrix **M** which governs the propagation of waves in a cold plasma parallel to a **B** field is given by

$$M = \begin{bmatrix} \varepsilon_1 - N^2 & -i\varepsilon_2 & o \\ i\varepsilon_2 & \varepsilon_2 - N^2 & o \\ o & o & \varepsilon_3 \end{bmatrix}$$

where N is the refractive index along the direction of propagation and ε_1, ε_2 and ε_3 are given below.

Determine the polarisation and dispersion relationships of all modes propagating parallel to **B**. [6]

Describe an experiment which measures the Faraday rotation of plane polarised radiation in a dielectric. [4]

Consider a right circular polarised wave with electric field amplitude $\sqrt{2}E_R$ and wave number k_R and a left circular polarised wave with elecric field amplitude $\sqrt{2}E_L$ and wave vector k_L both of frequency ω, travelling through a dielectric in the z direction parallel to **B**.

Show that for a plane polarised wave the ratio $\dfrac{E_x}{E_y} = \cot(k_L - k_R)\dfrac{z}{2}$ applies. [7]

What length of dielectric is required to produce a rotation of the plane polarised wave by 90°? [3]

You may assume:

$$\varepsilon_1 = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}$$

$$\varepsilon_2 = \frac{\omega_{ce}}{\omega} \cdot \frac{\omega_{pe}^2}{\omega_{ce}^2 - \omega^2}$$

$$\varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2}$$

where ω_{pe} and ω_{ce} are the plasma and cyclotron frequencies respectively.