UNIVERSITY OF LONDON

M.Sci DEGREE 2000

PHYS4461: PLASMA PHYSICS

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Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Symbols used, with values where appropriate:

m_e	mass of electron $= 9.1 \times 10^{-31}$ kg
m_p	mass of proton $= 1.7 \times 10^{-27}$ kg
$\underline{u}$	average drift velocity
$v_\perp$	particle speed in a plane perpendicular to $\underline{B}$
a	radius of gyration
q	charge on a body
e	charge on a proton $= 1.6 \times 10^{-19}$ C
k	the Boltzmann constant $= 1.4 \times 10^{-23}$ $J\,K^{-1}$
$\underline{J}$	current density
$\underline{E}$	electric field vector (magnitude E)
$\underline{B}$	magnetic induction vector (magnitude B)
ρ	plasma density
p	plasma pressure
ν	collision frequency between two species of particles
ϵ_o	permittivity of free space $= 8.85 \times 10^{-12}$ F m^{-1}
μ_o	permeability of free space $= 4\pi \times 10^{-7}$ H m^{-1}

[Part marks]

1. Derive expressions for the angular frequency, the radius of gyration (in terms of mass and velocity) and the drift-velocity of a charged particle moving in a crossed electric and magnetic field. Sketch the motion relative to the two fields, highlighting any feature which is associated with the sign of the charge on the particle. **[8]**

   Recast the expression derived earlier so that the radius of gyration is specified in terms of the kinetic energy associated with the orbital motion of the particle. Use this to determine the radius of the trajectory of a 100eV electron where the magnetic induction is 2×10^{-5} tesla. (1eV $= 1.6 \times 10^{-19}$ J.) **[5]**

   Given that the magnetic moment of a single current-loop is $\underline{\mu} = (I\underline{S})$, where its enclosed area is $\underline{S}$ m^2 and it carries I amp, determine μ for a charged particle undergoing circular motion within a uniform magnetic field. (The kinetic energy of the particle and the magnetic induction should appear in the expression.) Use your result to describe what happens to the particle as the flux density changes slowly (assuming the magnetic moment and total kinetic-energy to be invariant), and hence explain reflection of charged particles at a magnetic mirror. **[7]**

2. Assuming conditions remain steady, use the momentum transfer equation for an electron,

$$m_e\dot{\underline{u}} = -e(\underline{E} + \underline{u} \times \underline{B}) - m_e\nu\underline{u} \ ,$$

to derive an expression for the dc conductivity of the plasma, σ_o, which appears in the generalized form of Ohm's law,

$$\underline{J} = \sigma_o(\underline{E} + \underline{u} \times \underline{B}).$$

[4]

Employing a cartesian coordinate system with the z-axis parallel to the applied magnetic field, separate out the three vector components from the expression just quoted to show how each component of the current-density relates to the various components of the applied electric field. Derive the electrical conductivity corresponding to each axis of the coordinate system. **[8]**

Discuss these results qualitatively, stressing why the cyclotron-frequency and collision-frequency of the charged carriers are pertinent factors. **[4]**

Explain how the electrical conductivity along one axis perpendicular to the magnetic field becomes enhanced when flow of current along the other axis perpendicular to this magnetic field is inhibited. **[4]**

3. Consider an isothermal, non-ionized gas which has developed a weak density gradient. Using the mass- and momentum-transfer equations

$$\dot{\rho} + \underline{\nabla} \cdot (\rho\underline{u}) = 0$$

$$\text{and} \qquad \dot{\underline{u}} + \underline{u} \cdot \nabla\underline{u} = -\frac{e}{m}(\underline{E} + \underline{u} \times \underline{B}) - \frac{1}{\rho}\underline{\nabla}p - \nu\underline{u},$$

derive the damped wave-equation which describes how the system relaxes. Examine the individual terms in this expression (using order-of-magnitude considerations) to determine conditions under which 'classical' diffusion results. You are required to identify the diffusion coefficient, and derive the characteristic time taken by the system to relax back to a uniform state. **[12]**

What differences arise when similar considerations are applied to a fully-ionized plasma? You are not expected to derive the appropriate equations, but should comment on why the simple 'free electron' diffusion coefficient is not generally valid. **[8]**

4. A small spherical test particle carrying positive charge Q is embedded in an otherwise neutral plasma of temperature T and electron number density n_e. Describe the charge-configuration which develops, and use the differential form of Gauss' law to show that

$$\nabla^2\phi = \frac{2}{\lambda_D^2}\phi,$$

where ϕ is the electrostatic potential towards the edge of the disturbed region, and λ_D is the Debye length. **[12]**

Why does a positive-ion sheath form around a conducting plate placed in a plasma? Provide a sketch of the current versus applied-voltage characteristic of a Langmuir probe formed by varying the potential on such a plate, and explain how the various features of the characteristic come about. **[8]**

5. (a) Discuss the temperature dependence of the various parameters which affect the energy-balance of a deuterium-tritium plasma, and explain what conditions are necessary if a low ignition temperature for thermonuclear reactions is to be obtained. **[10]**

How is the minimum useful confinement time for the plasma determined? Explain how this factor determines the dimensions and strength of the confining magnetic field. **[3]**

(b) How is energy believed to be stored in a magnetic field prior to the onset of flaring in the solar corona? Discuss the factors which keep the configuration stable during the initial build-up, and the subsequent changes which are thought to trigger rapid release of energy. **[7]**

END OF PAPER