UNIVERSITY OF LONDON

M.Sci DEGREE 2001

PHYS4461: PLASMA PHYSICS

Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Symbols used, with values where appropriate:

m_e	mass of electron = 9.1×10^{-31} kg
m_p	mass of proton = 1.7×10^{-27} kg
$\underline{u}$	average drift velocity
$v_\perp$	particle speed in a plane perpendicular to $\underline{B}$
n_o	particle number density where the plasma is undisturbed
a	radius of gyration
q	charge on a body
e	charge on a proton = 1.6×10^{-19} C
k	the Boltzmann constant = $1.4 \times 10^{-23}\ J\ K^{-1}$
$\underline{E}$	electric field vector (magnitude E)
$\underline{B}$	magnetic induction vector (magnitude B)
ν	collision frequency between two species of particles
ϵ_o	permittivity of free space = 8.85×10^{-12} F m^{-1}
μ_o	permeability of free space = $4\pi \times 10^{-7}$ H m^{-1}

[Part marks]

1. Derive expressions for the angular frequency, the drift-velocity, and the radius of gyration of a charged particle moving in a crossed electric and magnetic field. Provide sketches to illustrate these motions relative to the two fields, commenting on any differences arising from the sign of the charge on the particle. **[11]**

   Use the result to derive the particle drift-velocities in the two following cases:

   a) the electric field is replaced by a gravitational field.

   b) there is no electric field but the magnetic induction has a gradient $(\nabla_\perp B)$ in one dimension across the orbit. You may assume the net force averaged over an orbit to be $0.5 q a v_\perp (\nabla_\perp B)$. **[4]**

   Specify in each case, using diagrams, how the drift direction depends on charge polarity. **[2]**

   An electron travels at 5×10^7 m s^{-1} in a plane perpendicular to a magnetic field ($B \sim 1$ T). Assuming uniform conditions, calculate its radius of gyration and the drift velocity imposed by a gradient of 10^{-2} T m^{-1} in the induction. **[3]**

2. Explain what is meant by the *Debye-length* and the *Plasma-frequency* of a fully ionized plasma. Derive expressions for each of these parameters. **[8]**

A non-magnetic plasma is perturbed by a wave having angular frequency ω, which propagates along the x-axis. Given the momentum transfer equation for electrons,

$$m_e \underline{\dot{u}} = -e(\underline{E} + \underline{u} \times \underline{B}) - m_e \nu \underline{u} \ ,$$

derive the dispersion relations for the longitudinal and transverse components of the electric vector for the case where collisions between particles can be disregarded. Discuss the implications of these results regarding wave propagation at different frequencies. **[9]**

What criterion determines whether collisions can be disregarded? Describe, with the aid of a diagram, how the damping of a wave varies with collision frequency. **[3]**

3. Under what circumstances can magnetohydrodynamic (MHD) approximations be utilized to study a plasma? Quote the basic linearized expressions which can then be employed, and use them to derive the force per unit volume on a magnetized plasma in terms of magnetic pressure and tension in the field lines. **[7]**

Describe how these concepts of magnetic pressure and magnetic tension allow one to visualize interactions between plasma and magnetic flux. **[2]**

Use the MHD equations to show how the conductivity of a drifting plasma determines how the magnetic field which threads it alters with time. Explain the significance of *magnetic viscosity* and *magnetic Reynolds number*, which appear, and derive the *diffusion time* which characterizes relative motion when the Reynolds number is much less than unity. **[8]**

The Sun has a radius of 7×10^8 m and an electrical conductivity of order 5×10^7 (ohm m)$^{-1}$. For how long might it be expected to retain its internal magnetic field? **[3]**

The following identities may be of use, where $\underline{V}$ is a general vector quantity:

$$(\underline{\nabla} \times \underline{V}) \times \underline{V} = (\underline{V}.\underline{\nabla})\underline{V} - \frac{\underline{\nabla} V^2}{2}$$

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{V}) = \underline{\nabla}(\underline{\nabla}.\underline{V}) - \nabla^2 \underline{V}$$

4. (a) A cylindrical magnetic flux-tube, filled with low-density plasma, is twisted n turns about its longitudinal axis. Taking the length of the tube to be L and the radius to be R, determine the additional energy introduced by the twisting process. You may assume that the longitudinal component of the magnetic flux-density is maintained at B_z tesla. **[13]**

Substitute the values n = 1, R = 10^6m and L = 10^7m into the expression derived, in order to determine the value of B_z required to store the 10^{22}J which is released in a modest solar flare. **[2]**

(b) Solar flares are widely considered to arise when two magnetic flux tubes with anti-parallel fields come into contact. Discuss the characteristics of the current sheet which forms between these two structures, paying particular attention to plasma conditions and the configuration of any local electric current. **[5]**

5. Derive the relation (Bennett's relation) between the number-density and temperature of material in an unmagnetized linear pinch and the current flowing along the column. The general expression for magnetic pressure is $B^2/(2\mu_o)$ N m^{-2}, and the following identity for a cylindrical system may be of use: **[11]**

$$\underline{\nabla} \times \underline{V} \equiv \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi) \quad \text{where} \quad \underline{V} \equiv \underline{V}(\phi)$$

Describe qualitatively how sausage and kink instabilities develop in a pinched system, and point out how the incidence of these deformations can be reduced. What is the current needed to 'pinch' a cylindrical conducting column of radius 0.1m, composed of 10^8 K plasma, when the number density is 10^{14} m^{-3}? **[5]**

Outline the main geometry of a magnetic-mirror device designed to enclose hot plasma, describe how it functions and explain why it is not ideal for use in a commercial fusion reactor. **[4]**

END OF PAPER