UNIVERSITY OF LONDON

M.Sci DEGREE 1999

PHYS4461: PLASMA PHYSICS

Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Symbols used, with values where appropriate:

m_e	mass of electron = 9.1×10^{-31} kg
m_p	mass of proton = 1.7×10^{-27} kg
$\underline{u}$	average drift velocity
$v_\perp$	particle speed in a plane perpendicular to $\underline{B}$
n_o	particle number density where the plasma is undisturbed
a	radius of gyration
q	charge on a body
e	charge on a proton = 1.6×10^{-19} C
k	the Boltzmann constant = 1.4×10^{-23} $J\,K^{-1}$
$\underline{E}$	electric field vector (magnitude E)
$\underline{B}$	magnetic induction vector (magnitude B)
ρ	plasma density
p	plasma pressure
ν	collision frequency between two species of particles
ϵ_o	permittivity of free space = 8.85×10^{-12} F m^{-1}
μ_o	permeability of free space = $4\pi \times 10^{-7}$ H m^{-1}

[Part marks]

1. Derive expressions for the radius of gyration, the angular frequency, and the drift-velocity of a charged particle moving in a crossed electric and magnetic field. Provide sketches to illustrate these motions relative to the two fields, commenting on any differences arising from the sign of the charge on the particle. **[11]**

   Use the result to derive the particle drift-velocities in the two following cases:

   a) the electric field is replaced by a gravitational field.

   b) there is no electric field but the magnetic induction has a gradient $\nabla_\perp B$ in one dimension across the orbit. (You may assume the net force averaged over an orbit to be $0.5 q a v_\perp \nabla_\perp B$.) **[4]**

   Specify, using diagrams, how the drift direction depends on charge polarity in each case, and explain qualitatively how this motion comes about. **[2]**

   An electron travels at 5×10^7 m s^{-1} in a plane perpendicular to a magnetic field ($B \sim 1$ T). Calculate its radius of gyration, assuming uniform conditions, and the drift velocity imposed by a gradient of 10^{-2} T m^{-1} in the induction. **[3]**

2. How do electrons and positive ions interact when plasma oscillations develop? Derive a general expression for the plasma frequency, and estimate the value for a plasma having electron number density $10^{14}\ \mathrm{m}^{-3}$. **[4]**

Explain the significance of the Debye length, derive a general expression for this parameter, and deduce the value for the plasma just specified when the temperature is 10^6 K. **[4]**

The momentum transfer equation for electrons is

$$m_e \dot{\underline{u}} = -e(\underline{E} + \underline{u} \times \underline{B}) - m_e \nu \underline{u} \ .$$

An electromagnetic wave which has angular frequency ω is directed along the x-axis through a non-magnetic plasma. Derive the dispersion relations for the longitudinal and transverse components of the electric vector, assuming that collisions between particles can be disregarded. Discuss the implications of these results regarding wave propagation at different frequencies. **[12]**

3. What broad conditions must apply before magnetohydrodynamic (MHD) approximations can be utilized to study a plasma? Quote the basic linearized expressions employed in these circumstances, and use them to derive the force per unit volume on a magnetized plasma in terms of magnetic pressure and tension in the field lines. **[7]**

Describe how these concepts of magnetic pressure and magnetic tension allow one to visualize interactions between plasma and magnetic flux. **[2]**

Use the MHD equations to show how the conductivity of a drifting plasma determines how the magnetic field which threads it alters with time. Explain the significance of *magnetic viscosity* and *magnetic Reynolds number*, which appear, and derive the *diffusion time* which characterizes relative motion when the Reynolds number is much less than unity. **[8]**

Plasma having an electric conductivity of $10^8\ (\mathrm{ohm\ m})^{-1}$ needs to be held in a magnetic doughnut for about one second. Determine the minimum radius of the magnetic flux tube needed. **[3]**

The following identities may be of use, where $\underline{V}$ is a general vector quantity:

$$(\underline{\nabla} \times \underline{V}) \times \underline{V} = (\underline{V}.\underline{\nabla})\underline{V} - \frac{\underline{\nabla} V^2}{2}$$

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{V}) = \underline{\nabla}(\underline{\nabla}.\underline{V}) - \nabla^2 \underline{V}$$

4. Use the mass- and momentum-transfer equations

$$\dot{\rho} + \underline{\nabla}.(\rho \underline{u}) = 0$$

and $$\dot{\underline{u}} + \underline{u}.\nabla \underline{u} = -\frac{e}{m}(\underline{E} + \underline{u} \times \underline{B}) - \frac{1}{\rho}\,\underline{\nabla} p - \nu \underline{u}$$

to derive the damped wave-equation which describes how an isothermal, non-ionized gas responds to the development of a weak density gradient. Examine the individual terms in this expression (using dimensional analysis) to determine conditions under which 'classical' diffusion results. You are required to identify the diffusion coefficient, and derive the characteristic time taken by the system to relax back to a uniform state. **[9]**

What differences arise when similar considerations are applied to a fully-ionized plasma? You are not expected to derive the appropriate equations, but should comment on why the simple 'free electron' diffusion coefficient is not generally valid. **[7]**

An atmosphere is composed of neutrons, and another of (hypothetical) neutral particles having electron-mass. Derive scale-heights for the two cases, assuming the gravitational acceleration to be $g \sim 270$ m s^{-2}, which applies to the solar corona, and the temperature is 10^6 K. How is the mixture of electrons and ions distributed with height in the corona in practice? **[4]**

5. Derive the relation (Bennett's relation) between the temperature of material in an unmagnetized linear pinch and the current flowing along the column. The general expression for magnetic pressure is $B^2/(2\mu_o)$ N m^{-2}, and the following identity for a cylindrical system may be of use: **[12]**

$$\underline{\nabla} \times \underline{V} \equiv \frac{1}{r}\frac{\partial}{\partial r}(rV_\phi) \quad \text{where} \quad \underline{V} \equiv \underline{V}(\phi)$$

Describe qualitatively how sausage and kink instabilities develop in a pinched system, and point out how the incidence of these deformations can be reduced. What is the current needed to 'pinch' a cylindrical conducting column of radius 0.01m, composed of 10^8 K plasma, when the number density is 10^{14} m^{-3}? **[5]**

Outline the main geometry of a magnetic mirror device designed to enclose hot plasma, describe how it functions and explain why it is not ideal for use in a commercial fusion reactor. **[3]**

END OF PAPER