

Answer TWO questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Symbols used, with values where appropriate:

m_e	mass of electron = 9.1×10^{-31} kg
\underline{u}	average drift velocity
e	charge on a proton = 1.6×10^{-19} C
\underline{E}	electric field vector (magnitude E)
\underline{B}	magnetic induction vector (magnitude B)
\underline{J}	current density
ν	collision frequency between two species of particles
μ_o	permeability of free space = $4\pi \times 10^{-7}$ H m ⁻¹

[Part
marks]

1. Derive expressions which specify the trajectory of a charged particle in crossed electric and magnetic fields, where both fields are uniform. You are expected to determine the radius of gyration, the angular frequency, and the drift velocity. Provide diagrams to illustrate how the drift direction depends on charge polarity. [8]

Modify the expression derived earlier so that the radius of gyration is specified in terms of the kinetic energy associated with the orbital motion of the particle. Use this to determine the radius of the trajectory of a 1000eV electron in the 5 tesla magnetic field of a thermonuclear reactor. (1eV = 1.6×10^{-19} J.) [5]

Given that a current-loop of area \underline{S} m² carrying I amp has a magnetic moment $\underline{\mu} = (I\underline{S})$, determine this latter parameter for a charged particle undergoing circular motion within a uniform magnetic field. (The kinetic energy of the particle and the magnetic induction should appear in the expression.) Use your result to describe what happens to the particle as the flux density changes slowly (assuming the magnetic moment and total kinetic-energy to be invariant), and hence explain reflection of charged particles at a magnetic mirror. [7]

2. Given the momentum transfer equation for an electron,

$$m_e \dot{\underline{u}} = -e(\underline{E} + \underline{u} \times \underline{B}) - m_e \nu \underline{u} ,$$

derive an expression for the dc conductivity of the plasma, σ_o , which appears in the generalized form of Ohm's law,

$$\underline{J} = \sigma_o(\underline{E} + \underline{u} \times \underline{B}).$$

[4]

Using a cartesian coordinate system with the z-axis parallel to the applied magnetic field, separate out the three vector components from this latter relation, to show how each component of the current-density is determined by the various components of the applied electric field. Derive the electrical conductivity corresponding to each axis of the coordinate system.

[10]

Discuss these results qualitatively, stressing why the cyclotron-frequency and collision-frequency of the charged carriers are pertinent factors.

[3]

Explain how the electrical conductivity along one axis perpendicular to the magnetic field can become greatly enhanced when flow of current along the other axis perpendicular to this magnetic field is inhibited.

[3]

3. (a) Describe the general structure of a Tokamak reactor and explain how it functions. In doing so, you should highlight the advantageous features of the magnetic configuration employed.

[10]

Explain how values may be derived for *the dimensions of a Tokamak* and for *the magnetic induction employed*, assuming you are given the temperature and density of the plasma to be confined and the power generated per unit volume.

[3]

(b) Solar flares are said to arise when two magnetic flux-tubes with anti-parallel fields come into contact. Outline the characteristics of the current-sheet which forms between these two structures, giving attention to plasma conditions and the local electric current.

[4]

Assume the boundary layer to be $L \sim 100\text{m}$ thick, when the electrical conductivity of the plasma is $\sigma_o \sim 10^7 (\text{ohm} - \text{m})^{-1}$. Comment on whether diffusion of magnetic flux across the sheet is rapid enough to explain flaring, given that the characteristic timescale for this process is $\tau \sim L^2 \sigma_o \mu_o$ seconds.

[3]