

UNIVERSITY OF LONDON

M.Sci DEGREE 1997

PHYS4460: PLASMA PHYSICS

Answer TWO questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Symbols used, with values where appropriate:

m_e	mass of electron = 9.1×10^{-31} kg
\underline{u}	average drift velocity
e	charge on a proton = 1.6×10^{-19} C
k	the Boltzmann constant = 1.4×10^{-23} J K ⁻¹
\underline{E}	electric field vector (magnitude E)
\underline{B}	magnetic induction vector (magnitude B)
ν	collision frequency between two species of particles
ϵ_o	permittivity of free space = 8.85×10^{-12} F m ⁻¹
μ_o	permeability of free space = $4\pi \times 10^{-7}$ H m ⁻¹

[Part
marks]

1. How do electrons and positive ions interact when plasma oscillations develop? Derive a general expression for the plasma frequency, and estimate the value for a plasma having electron number density 10^{16} m⁻³. [4]

Explain the significance of the Debye length, derive a general expression for this parameter, and deduce the value for the plasma just specified when the temperature is 10^8 K. [4]

The momentum transfer equation for electrons is

$$m_e \dot{\underline{u}} = -e(\underline{E} + \underline{u} \times \underline{B}) - m_e \nu \underline{u} .$$

A wave which has angular frequency ω is directed along the x-axis through a non-magnetic plasma. Derive the dispersion relations for the longitudinal and transverse components of the electric vector, assuming that collisions between particles can be disregarded. Discuss the implications of these results regarding wave propagation at different frequencies. [12]

2. What broad conditions must apply before magnetohydrodynamic (MHD) approximations can be utilized to study a plasma? Quote the basic linearized expressions used in these circumstances, and use them to derive the force per unit volume on a magnetized plasma in terms of magnetic pressure and tension in the field lines. [7]

Describe how these concepts of magnetic pressure and magnetic tension allow one to visualize interactions between plasma and magnetic flux. [2]

Use the MHD equations to show how the conductivity of a drifting plasma determines how the magnetic field which threads it alters with time. Explain the significance of *magnetic viscosity* and *magnetic Reynolds number*, which appear, and derive the *diffusion time* which characterizes relative motion when the Reynolds number is much less than unity. [8]

Determine whether the Sun retains the internal magnetic field with which it formed about 5×10^9 years ago. This body has a radius of 7×10^8 m and an electrical conductivity of order 5×10^7 (ohm m) $^{-1}$. [3]

The following identities may be of use, where \underline{V} is a general vector quantity:

$$\begin{aligned}(\underline{\nabla} \times \underline{V}) \times \underline{V} &= (\underline{V} \cdot \underline{\nabla}) \underline{V} - \frac{\underline{\nabla} V^2}{2} \\ \underline{\nabla} \times (\underline{\nabla} \times \underline{V}) &= \underline{\nabla}(\underline{\nabla} \cdot \underline{V}) - \nabla^2 \underline{V}\end{aligned}$$

3. (a) Derive the relation (Bennett's relation) between the temperature of material in an unmagnetized linear pinch and the current flowing along the column. The general expression for magnetic pressure is $B^2/(2\mu_o)$ N m $^{-2}$, and the following identity may be of use: [8]

$$\underline{\nabla} \times \underline{V} \equiv \frac{1}{r} \frac{\partial}{\partial r} (rV_\phi) \quad \text{where} \quad \underline{V} \equiv \underline{V}(\phi)$$

Describe qualitatively the factors leading to the growth of sausage and kink instabilities in a pinched system, and point out how the incidence of these deformations can be reduced. What is the current needed to 'pinch' a cylindrical conducting column of radius 0.01m, composed of million-degree plasma, when the number density is 10^{14} m $^{-3}$. [4]

- (b) Discuss the temperature dependence of the various parameters which affect the energy-balance of a deuterium-tritium plasma, and explain what conditions are necessary if a low ignition temperature for thermonuclear reactions is to be obtained. [8]