**Answer TWO questions.**

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

# You may assume the following data:

| Particle name | Approx.Mass |
|---|---|
| Z | 91 GeV |
| W | 80 GeV |
| $\gamma$ | 0 GeV |
| gluon | 0 GeV |
| proton | 1 GeV |
| $\pi$ | 0.14 GeV |

## Mandelstam variables:

For a two-body scattering $A + B \to C + D$, the Mandelstam variables are given by:

$$\begin{aligned} s &= (p_A + p_B)^2 \\ t &= (p_A - p_C)^2 \\ u &= (p_A - p_D)^2 \end{aligned}$$

where $p_A$ is the four-momentum of particle $A$, and so on.

## Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

## Some constants:

$\hbar = 6.582 \times 10^{-25}$ GeV s.
$c = 2.998 \times 10^{8}$ m s$^{-1}$.

[Part marks]

1. The Dirac equation can be written

$$(\not{p} - m)\psi = 0$$

where $\not{p} = \gamma^\mu p_\mu$, $\psi$ is the four-component free particle spinor and $m$ is the particle mass. Show that if $m = 0$, the Dirac equation may be rewritten as two decoupled equations for a pair of two-component spinors. Work in the Weyl representation for the $\gamma$-matrices, that is:

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

[4]

The helicity operator is $\boldsymbol{\sigma} \cdot \boldsymbol{p}$. Show that the solutions of one equation describe a negative helicity particle and a positive helicity antiparticle, and the solutions of the other describe a positive helicity particle and a negative helicity antiparticle, and say which is which. [3]

Give an expression for the weak current $J^\mu$ in terms of the $\gamma$ matrices, $\psi$ and $\bar{\psi}$. Show that only one helicity of a massless neutrino participates in weak interactions and say which helicity this is. [4]

A number of $\mu^-$ decay at rest with their spins all pointing in a given direction. What is the most likely angle between the direction of the highest energy $e^-$ from the decay and the direction in which the $\mu^-$ spins were pointing and why?. [5]

Draw the leading order diagram(s) for $\nu_e$-quark scattering. Which process has the larger cross section, $\nu_e$-proton or $\bar{\nu}_e$-proton, and why? [4]

2. What is meant by the mass eigenstate and the weak eigenstate of a particle? If neutrinos have masses, it is possible that the mass eigenstates are not equal to the weak interaction eigenstates. By writing the weak eigenstates $\nu_e, \nu_\mu$ in terms of hypothetical mass eigenstates $\nu_1, \nu_2$ and a mixing angle $\alpha$,

$$\begin{aligned} \nu_e &= \nu_1 \cos\alpha + \nu_2 \sin\alpha \\ \nu_\mu &= -\nu_1 \sin\alpha + \nu_2 \cos\alpha \end{aligned}$$

derive an expression for the probability that a $\nu_e$ produced at time $t = 0$ is found as a $\nu_\mu$ at time $t$. (Assume that the $\nu_\tau$ does not mix). [6]

Describe briefly an existing piece of data which could be interpreted as evidence for neutrino oscillations. [3]

Experiments designed to detect neutrinos are generally very large and constructed deep underground (or under water). Why? [3]

An experiment sits 1000 km away from a source of electron neutrinos with energy $\approx 1$ GeV. If the mixing is as above and $\alpha = \pi/4$ and the experiment sees 50% muon neutrinos and 50% electron neutrinos, what is the smallest possible mass difference between the electron neutrino and the muon neutrino? You may assume that the mass of the neutrino is much smaller than its energy. [5]

How could a $\nu_\mu$ interaction be distinguished from a $\nu_e$ interaction in the experiment?. [3]

3. Explain qualitatively the property of the strong interaction which is responsible for the fact that the strong coupling constant $\alpha_s$ gets smaller at higher energies. [4]

Describe one piece of experimental evidence for the existence of gluons. [4]

The square of the invariant amplitude for unpolarised scattering $e^+e^- \to e^+e^-$ at $\sqrt{s} = 30$ GeV is:

$$|\mathcal{M}|^2 = 2e^4 \left( \frac{s^2 + u^2}{t^2} + \frac{2u^2}{st} + \frac{u^2 + t^2}{s^2} \right)$$

Draw the Feynman diagrams for this process and associate them with the different terms in the cross section above. [4]

Give an expression for the square of the leading order invariant amplitude for unpolarised scattering $e^+q \to e^+q$, for momentum transfers much less than $(90 \text{ GeV})^2$. [4]

In a particular class of $e^+p \to e^+X$ events, the hadronic system consists of two high transverse energy jets and a high transverse energy $e^+$. Draw the leading order Feynman diagrams for this process and suggest how the gluon content of the proton might be measured in an electron-proton collider. [4]



**END OF PAPER**