UNIVERSITY OF LONDON
(University College London)

MSci. Degree 1997

PHYS4445: ADVANCED PARTICLE PHYSICS

Answer **TWO** questions

The numbers in square brackets at the right-hand edge of the paper indicate the provisional allocation of maximum marks for each subsection of a question.

You may assume the following data:

(a) Quark quantum numbers

The quantum numbers *Q* (electric charge), *S* (strangeness), *C* (charm), *B* (beauty), T (truth), I_3 *(third component of isospin) and the approximate masses in* GeV/c^2 *of the d, u, s, c, b* and *t* quarks (all five have baryon number 1/3 and positive parity) are:

Name	***Symbol***	***Mass***	***Q***	***S***	***C***	***B***	***T***	**I_3**
Down	*d*	0.35	Ð1/3	0	0	0	0	Ð1/2
Up	*u*	$m_u \approx m_d$	2/3	0	0	0	0	1/2
Strange	*s*	0.5	Ð1/3	Ð1	0	0	0	0
Charmed	*c*	1.5	2/3	0	1	0	0	0
Bottom	*b*	4.5	Ð1/3	0	0	Ð1	0	0
Top	*t*	180	2/3	0	0	0	1	0

(b) Quark colour charges

Values of the colour hypercharge Y^C and colour isospin I_3^C for quarks. All these values reverse sign for antiquarks.

Colour state	**Y^C**	**I_3^C**
r	1/3	1/2
g	1/3	Ð1/2
b	Ð2/3	0

(c) Quark compositions of hadrons

$$K^+ = u\bar{s},\ \mathrm{p}^+ = u\bar{d},\ \mathrm{p}^0 = u\bar{u} \text{ or } d\bar{d},\ p = uud,\ n = uud$$

(d) Numerical values

$$m_K = 494\,MeV/c^2,\ m_e = 0.51\,MeV/c^2,\ m_{\mathrm{m}} = 106\,MeV/c^2$$

$$B(Z^0 \to \ell^+\ell^-) = 0.034 \quad (\ell = e,\ \mathrm{m}); \qquad G_F/(\hbar c)^3 = 1.17\text{x}10^{-5}\,GeV^{-2}$$

PLEASE TURN OVER

1. Write down one reaction which provides unambiguous experimental evidence for the existence of weak neutral currents.

[2 marks]

Draw Feynman diagrams for the basic Z^0-lepton vertices. Use lepton quark symmetry to deduce the corresponding Z^0-quark vertices in the case of two generations of quarks:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

where

$$d' = d\cos\theta_C + s\sin\theta_C$$

and

$$s' = -d\sin\theta_C + s\cos\theta_C$$

with θ_C the Cabibbo angle. Hence deduce that weak neutral currents conserve flavour.

[8 marks]

Draw a Feynman diagram to show why this result forbids the decay $K^+ \to \pi^+ + \nu_\mu + \bar{\nu}_\mu$ as a first-order weak interaction.

[2 marks]

Draw a Feynman diagram involving the exchange of two charged *W*-bosons which would allow this K^+ decay. Hence, using simple dimensional arguments, make an order-of-magnitude estimate of the ratio of decay rates:

$$R \equiv \frac{\Gamma(K^+ \to \pi^+ + \bar{\nu}_\mu + \nu_\mu)}{\Gamma(K^+ \to \pi^0 + \mu^+ + \nu_\mu)}$$

[8 marks]

2. State *briefly* one piece of evidence that quarks possess the attribute of *colour*.

[3 marks]

Show, by considering quark-quark scattering, that the colour hypothesis leads to the conclusion that gluons couple to other gluons.

[2 marks]

CONTINUED

What is the hypothesis of *colour confinement* in terms of the colour charge operators $\hat{F}_i\,(i = 1,\ldots,8)$?

[2 marks]

Consider the combination $q^m \bar{q}^n$ of *m* quarks and *n* antiquarks with a colour wavefunction

$$r^{a} g^{b} b^{g} \bar{r}^{\bar{a}} \bar{g}^{\bar{b}} \bar{b}^{\bar{g}}$$

where r^{a} etc means there are a quarks in the ÔredÕ state etc. and

$$m = a + b + g > n = \bar{a} + \bar{b} + \bar{g}\ .$$

Use the colour confinement condition for the colour charge operators $\hat{I}_3^{\,C} = \hat{F}_3$ and $\hat{Y}^C = \frac{2}{\sqrt{3}}\hat{F}_8$ to show that: (a) the combinations $meson = q\bar{q}$, and $baryon = qqq$ used in the simple quark model are allowed; and (b) hadrons with fractional electric charge are forbidden.

[5 marks]

In the quark model, the most general colour wavefunction for a baryon is

$$c = a_1 r_1 g_2 b_3 + a_2 g_1 r_2 b_3 + a_3 b_1 r_2 g_3 + a_4 b_1 g_2 r_3 + a_5 g_1 b_2 r_3 + a_6 r_1 b_2 g_3$$

where the $a_j (j = 1,\ldots,6)$ are constants and

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

are the colour states of a quark. In the wavefunction, $r_1 (g_2$ etc$)$ means quark 1 is in a colour state ÔredÕ (quark 2 is in a colour state ÔgreenÕ etc). Find the restrictions on the coefficients implied by colour confinement for $i = 1$ where

$$\hat{F}_1 = \tfrac{1}{2} l_1 \qquad \text{and} \qquad l_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

[5 marks]

If, in addition, $a_2 = -a_3$ and $a_4 = -a_5$, show that c is consistent with the Pauli Principle applied to the quark model of baryons.

[3 marks]

PLEASE TURN OVER

3. Maxwell's equations for the electromagnetic fields **E** and **B** in free space are:

$$div\,\mathbf{B} = 0, \qquad curl\,\mathbf{E} = -\frac{\partial\,\mathbf{B}}{\partial t}, \qquad div\,\mathbf{E} = 0, \qquad curl\,\mathbf{B} = \frac{\partial\,\mathbf{E}}{\partial t}$$

Use the definitions

$$\mathbf{E} = -\nabla\phi - \frac{\partial\mathbf{A}}{\partial t}, \qquad \mathbf{B} = \nabla \wedge \mathbf{A}$$

to derive the wave equations for the scalar (ϕ) and vector (**A**) potentials and show that these equations are invariant under the gauge transformations

$$\phi \to \tilde{\phi} = \phi + \frac{\partial f}{\partial t}, \qquad \mathbf{A} \to \tilde{\mathbf{A}} = \mathbf{A} - \nabla f$$

where $f(\mathbf{x}, t)$ is an arbitrary scalar function.
[You may assume the vector identity: $\nabla \wedge \nabla \wedge \mathbf{A} = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A}$]

[3 marks]

By choosing a gauge which satisfies the Lorentz condition

$$\frac{\partial\phi}{\partial t} + \nabla\cdot\mathbf{A} = 0 \quad .$$

and comparing the transformed equations with the Klein-Gordon equation

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\psi + m^2\psi = 0 \quad ,$$

deduce that gauge invariance implies that photons have zero mass.

[2 marks]

What would be an experimental consequence of $m_\gamma \neq 0$? **[2 marks]**

Why is it necessary to introduce a Higgs field into the Standard Model and what are the major consequences of doing so?

[4 marks]

A Higgs boson with mass $M_H = M_Z$ will decay to fermion-antifermion pairs $f\bar{f}$ with a rate proportional to

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$$\alpha_{Hff} \equiv \frac{G_F\, m_f^{\,2}}{2\pi}$$

where G_F is the Fermi weak coupling constant. Use this result to estimate the branching ratios of the Higgs boson to e^+e^- and $\mu^+\mu^-$ pairs.

[3 marks]

Hence justify the neglect of the exchange of a Higgs boson when analysing the reaction $e^+e^- \to \mu^+\mu^-$ at $E_{CM} \approx M_Z$. You may assume the Breit-Wigner form

$$\sigma\left(e^+e^- \to \mu^+\mu^-\right) = \frac{12\pi\, M_X^{\,2}}{E_{CM}^{\,2}}\left[\frac{\Gamma(e^+e^- \to X)\Gamma(X \to \mu^+\mu^-)}{(E_{CM}^{\,2} - M_X^{\,2})^2 + M_X^{\,2}\Gamma_X^{\,2}}\right]$$

for the production of a state X in the reaction $e^+e^- \to \mu^+\mu^-$.

[4 marks]

Why is the decay mode $H^0 \to f\bar{f}$ unlikely to be a very useful one for detecting the Higgs boson for low masses $M_H < 2M_Z$ and in this case give one other final state which might be used?

[2 marks]

END OF PAPER