Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may assume the following data:

Particle name	Approx.Mass
c quark	1.5 GeV
b quark	5.0 GeV

Mandelstam variables: For a two-body scattering $1+2 \to 3+4$, the Mandelstam variables are given by:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}$$

where p_1 is the four-momentum of particle 1, and so on.

Differential cross section in two-body scattering: For $1+2 \to 3+4$ in the centre-of-mass frame, Fermi's Golden Rule gives:

$$\frac{d\sigma}{d\Omega} = \frac{1}{(8\pi)^2} \frac{|\mathcal{M}|^2}{E_{CM}^2} \frac{|\vec{p}_f|}{|\vec{p}_i|}$$

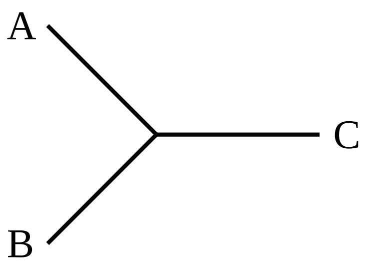
where $\mathcal{M}$ is the invariant amplitude, E_{CM} the centre-of mass energy, and $|\vec{p}_i|$ and $|\vec{p}_f|$ are the initial and final state momenta.

Euler-Lagrange equations: For a continuous system $\phi(x)$:

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

[Part marks]

1. Imagine a world in which there are just three kinds of particles: A, B and C. They all have spin 0 and each is its own antiparticle. There is only one primitive vertex by which the particles interact:



and the strength of the interaction is determined by a coupling constant g.

(a) Assuming that $m_A > m_B + m_C$, $A \to B + C$ is the most likely decay mode of A. If A is heavy enough what are the two next most likely decay modes? Draw the corresponding Feynman diagrams. [4]

(b) Draw the Feynman diagram(s) and determine the lowest order amplitude, $\mathcal{M}$, for the process $A + B \to A + B$; express it in terms of the Mandelstam variables. [7]

(c) Assuming that $m_A = m_B$ and $m_C = 0$, find the differential cross section for $A + B \to A + B$ in the centre-of-mass (CM) frame and express it in terms of the CM energy, E_{CM}, and the scattering angle, θ. You may assume that E_{CM} is high enough that approximations such as $E_A \approx |\vec{p}_A|$ can be made. [6]

(d) What are the dimensions of the differential cross section in natural units? What about those of the above amplitude, $\mathcal{M}$, and the coupling constant g? [3]

2. (a) Prove that the Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

describes particles with some spin. [7]

(b) The momentum space Dirac equation can be written as

$$(\not{p} - m)\psi = 0$$

where $\not{p} = \gamma^\mu p_\mu$, ψ is the four-component free particle spinor and m is the particle mass. By writing ψ in terms of a pair of two-component spinors, ϕ and χ, and using the Weyl representation of the γ matrices

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix},$$

show that for massless particles the above equation can be decomposed into independent equations for ϕ and χ, one describing negative helicity particles and positive helicity antiparticles and the other positive helicity particles and negative helicity antiparticles. [8]

(c) Give one example of experimental results that led particle physicists to believe (for several decades!) that neutrinos are massless. Describe briefly one recent experimental result that produced categoric evidence of the contrary. [5]

3. (a) Show that the Lagrangian density

$$\mathcal{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

leads to the Dirac equation for ψ and the adjoint Dirac equation for $\bar{\psi}$. [5]

(b) Show that $\mathcal{L}$ is invariant under global gauge transformations but not under local gauge transformations. [6]

(c) Local gauge invariance can be achieved by replacing the derivative ∂_μ with the "covariant derivative"

$$\mathcal{D}_\mu \equiv \partial_\mu - ieA_\mu$$

(e is a constant), as long as A_μ transforms in a certain way. Derive the transformation rules for A_μ and give its physical interpretation. [6]

(d) What would be the implications of adding a term of the form $\frac{1}{2}m_A^2 A^\mu A_\mu$ to the QED Lagrangian density? [3]

4. (a) Assuming that the Standard Model Higgs boson exists and $m_H = 120\,\text{GeV}$, draw the Feynman diagram for the Higgs production process with the highest cross section at the LHC. Which decay mode of the above produced Higgs will be searched for? Draw the corresponding Feynman diagram. What is the production-decay chain that will be searched for at the Tevatron and why is it different from the above? [7]

(b) Why do we consider muons instead of electrons for a future high energy circular lepton collider? Describe briefly three of the main difficulties in building such a muon collider. [8]

(c) Which experimental result primarily supports the current belief that there are no more than three generations of elementary particles? To what extent is this considered a "closed issue"? [5]

5. (a) Sketch the ratio

$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for centre-of mass energies from 2 GeV to 12 GeV. What is the most important information that R provides? Why not use $\sigma(e^+e^- \to e^+e^-)$ in the denominator of R? [7]

(b) Explain qualitatively the running of the coupling constant in QED. What is different in QCD and what behaviour does it lead to? [6]

(c) Show that in elastic electron-proton scattering

$$q \cdot p = -\frac{q^2}{2}\,,$$

where p is the proton's four-momentum and q is the four-momentum of the exchanged photon. Explain briefly (in words) the concept of Bjorken scaling. What is the meaning of the Bjorken variable x? Prove that $0 \leq x \leq 1$. [7]

 END OF PAPER