Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may assume the following data:

Particle name	Approx.Mass
Z	91 GeV
W	80 GeV
γ	0 GeV
gluon	0 GeV
proton	1 GeV
π	0.14 GeV
beauty quark	$\approx$ 5.0 GeV
charm quark	$\approx$ 1.5 GeV

Mandelstam variables:

For a two-body scattering $A + B \to C + D$, the Mandelstam variables are given by:

$$\begin{aligned} s &= (p_A + p_B)^2 \\ t &= (p_A - p_C)^2 \\ u &= (p_A - p_D)^2 \end{aligned}$$

where p_A is the four-momentum of particle A, and so on.

Euler-Langrange equations for a continuous system $\phi(x)$:

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

[Part marks]

1. Draw the leading order Feynman diagram(s) for the reaction $e^+e^- \to \mu^+\mu^-$. In each diagram, what is the squared four-momentum transferred by the propagator in terms of the Mandelstam variables? Give an expression for the propagator in each diagram, defining all terms you use. **[4]**

   Write down the Klein-Gordon equation. Show how a decaying free particle solution to the Klein-Gordon equation,

   $$\psi = Ne^{-ip\cdot x}e^{-\Gamma t/2}$$

   can be thought of as implying an imaginary term subtracted from the square of the mass, in the case that the mass of the particle $M \gg \Gamma$. **[5]**

   Use the expression for a massive boson propagator, and the solution above, to derive the Breit-Wigner formula for a resonance,

   $$\sigma \propto \frac{1}{(W-M)^2 + \Gamma^2/4}$$

   where W is the centre of mass energy and is close to M. (You may neglect the $q^\mu q^\nu$ term in the propagator). **[5]**

   Sketch the shape of the cross section for $\sigma(e^+e^- \to \mu^+\mu^-)$ in the range $0 < W < 100$ GeV and explain the main features. **[3]**

   Say briefly how a measurement of Γ for the Z at LEP can be used to calculate the number of generations. Outline the main assumptions and additional measurements that are required. **[3]**

2. The square of the invariant amplitude for unpolarized scattering $e^+e^- \to e^+e^-$ at $\sqrt{s} = 30$ GeV in terms of the Mandelstam variables is:

$$|\mathcal{M}|^2 = 2e^4 \left(\frac{s^2 + u^2}{t^2} + \frac{2u^2}{st} + \frac{u^2 + t^2}{s^2} \right)$$

Draw the Feynman diagrams for the amplitudes involved in this process. Associate the different terms in the cross section above with your diagrams. **[5]**

Draw the most important Feynman diagrams for unpolarized $e^+e^- \to \mu^+\mu^-$ and for $e^+e^- \to q\bar{q}$, also at 30 GeV. Write down the invariant amplitude squared for these processes in terms of the Mandelstam variables. **[5]**

The measured ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

provides evidence that quarks come in three colours. From the above results, explain why and estimate this ratio at 30 GeV and at 2 GeV. What is the major effect which would modify these answers? Estimate its size. **[5]**

Describe briefly what detector components would be needed in order to distinguish the three processes $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to$ hadrons from each other. Discuss how each event type would appear in these components. **[5]**

3. Proton anti-proton collisions take place at the Tevatron at a centre of mass energy of 2 TeV. Two processes which can take place are $u\bar{u}$ annihilation and $d\bar{d}$ annihilation. Say which makes the larger contribution to the proton-antiproton scattering cross section and why. **[2]**

Draw all the leading-order Feynman diagrams for the process $u\bar{u} \rightarrow b\bar{b}$. Arrange these diagrams in order of their expected relative rates (most common first, least common last) and give your reasons for this ordering. **[4]**

Draw a schematic diagram of a $p\bar{p} \rightarrow b\bar{b} + X$ event in a typical detector at the Tevatron, indicating the different components and showing what happens to the other quarks and antiquarks which were inside the colliding particles. How might the b and $\bar{b}$ quarks be identified in a detector? **[5]**

In a particular event, one of the beauty quarks travels at an angle of $\theta_1$with respect to the proton beamline, and the other travels at an angle of θ_2 with respect to the proton beamline. Both quarks have a transverse momentum of magnitude p_T. Write down the four-momentum of each of the quarks, defining the z axis to be along the proton beam direction and assuming both quarks have zero momentum along the y axis. **[4]**

If $\theta_1 = 30^o$, $\theta_2 = 45^o$ and $p_T = 50$ GeV and the scattering process was $u\bar{u} \rightarrow b\bar{b}$ with the u quark coming from the proton, calculate x, the fraction of the proton's momentum carried by the u quark (assume the incoming quarks have no transverse momentum or mass). **[5]**

4. At the HERA accelerator, 30 GeV unpolarized electrons collide with 920 GeV protons. In a particular class of event, electrons scatter from a proton at an angle θ and become neutrinos. Draw the leading order Feynman diagram(s) for the process e quark $\rightarrow \nu$ quark. Derive an expression for the square of the four-momentum carried by the propagator, q^2 , in terms of θ and the incoming and outgoing quark energies. (Neglect the electron and quark masses). **[7]**

The Dirac equation may be written

$$H\psi = (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m)\psi$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ can be represented as

$$\alpha_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$$

where the $\sigma_i, i = 1, 2, 3$ are the Pauli spin matrices. Using this, for the case $m = 0$ decompose the Dirac equation into decoupled equations for the two-component spinors ϕ and χ where

$$\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix}.$$

[3]

Show that if the helicity operator is $\boldsymbol{\sigma} \cdot \mathbf{p}$, the solutions of one equation describe a negative helicity particle and a positive helicity antiparticle, and the solutions of the other describe a positive helicity particle and a negative helicity antiparticle. **[3]**

Given that the weak charged current may be written as $J^\mu = \frac{1}{2}\bar{\psi}\gamma^\mu(1 - \boldsymbol{\sigma} \cdot \mathbf{p})\psi$, say how and why you expect the cross section for e^- quark $\rightarrow \nu$ quark to change if the electron beam changed from zero polarization to being 100% polarized, first to positive helicity and then to 100% negative helicity. **[4]**

How would the unpolarized cross section for e^- quark $\rightarrow \nu$ quark compare to that for e^+ quark $\rightarrow \overline{\nu}$ quark? **[3]**

5. Show that the Lagrangian density for an electron:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

is invariant under the transformation:

$$\psi(x) \rightarrow \psi(x)\exp(i\alpha)$$

where α is a constant. **[2]**

Show that the Lagrangian density is not invariant under this transformation if α is a function of x^μ. **[2]**

The invariance can be restored by replacing the derivative ∂_μ by the covariant derivative:

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

where e is a constant. What transformation properties must A_μ have to make this work? What is the physical interpretation of A_μ? **[5]**

Show that assigning a mass to A_μ would again break the symmetry. **[3]**

In words and/or equations, describe the difficulties which arise in the standard model from assigning a mass to the W and Z bosons, and how the Higgs mechanism attempts to resolve them. Draw a possible Feynman diagram for Higgs production in a proton-antiproton collision and say what the expected final state would be and why. **[8]**

END OF PAPER