Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may assume the following data:

Particle name	Approx.Mass
Z	91 GeV
W	80 GeV
γ	0 GeV
gluon	0 GeV
proton	1 GeV
π	0.14 GeV
beauty quark	≈ 5.0 GeV
charm quark	≈ 1.5 GeV

Mandelstam variables:

For a two-body scattering $A + B \to C + D$, the Mandelstam variables are given by:

$$\begin{aligned} s &= (p_A + p_B)^2 \\ t &= (p_A - p_C)^2 \\ u &= (p_A - p_D)^2 \end{aligned}$$

where p_A is the four-momentum of particle A, and so on.

Euler-Langrange equations for a continuous system $\phi(x)$:

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi/\partial x_\mu)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

[Part marks]

1. Draw the leading order Feynman diagram(s) for the reaction $e^+e^- \to b\bar{b}$. What is the squared four-momentum transferred by the propagator in terms of the Mandelstam variables? Give an expression for the propagator, defining all terms you use. **[5]**

   The PEP II collider at Stanford Linear Accelerator Lab in California collides 9 GeV electrons with 3.1 GeV positrons. Calculate the centre-of-mass energy. **[2]**

   Sketch the ratio

$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

   for centre-of-mass energies from 1 GeV to just above the PEP II centre of mass energy, and explain the main features. Why is the PEP II centre-of-mass energy chosen? **[5]**

   Describe in words (and diagrams if necessary) the processes undergone by beauty and anti-beauty quarks produced at PEP II after they are produced. **[3]**

   Say briefly how beauty quarks be identified and their lifetimes measured, and thus suggest a reason why PEP II uses asymmetric beams energies. **[5]**

2. In the quark parton model the hadron structure function $F_2(x, Q^2)$ is the sum of the quark momentum densities inside the hadron multiplied by the square of their charges. Define the Bjorken variable x and say what meaning it has within the quark parton model. Define what is meant by momentum density. **[4]**

   Sketch (and explain) the form you would expect for $F_2^{\text{proton}}(x)$ if the proton was:

   - An elementary particle
   - 3 free quarks
   - 3 quarks bound in a potential well
   - 3 quarks bound by QCD.

   [7]

   At the HERA accelerator, 30 GeV electrons collide with 920 GeV protons. In a particular class of event, electrons scatter from a proton at an angle θ. Draw the leading order Feynman diagram(s) for the process e quark $\rightarrow$ e quark. If the propagator carries four-momentum q, show that

   $$q^2 = -2E_A E_C(1 - \cos\theta)$$

   where E_A, E_C are the incoming and outgoing electron energies respectively (Neglect the electron and quark masses). **[6]**

   At low values of $|q^2|$, the process e quark $\rightarrow$ e quark can no longer be used to help calculate the electron-proton cross section. At roughly what value of $|q^2|$ does this happen. Why? **[3]**

3. Draw the leading order Feynman diagrams for neutrino-quark interactions for both electron and muon neutrinos. Outline the principle requirements of a neutrino detector and say how a ν_μ interaction be distinguished from a ν_e interaction in such an detector.. **[8]**

What is meant by the mass eigenstate and the weak interaction eigenstate of a particle? If neutrinos have masses, it is possible that the mass eigenstates are not equal to the weak interaction eigenstates. By writing the weak eigenstates ν_e, ν_μ in terms of hypothetical mass eigenstates ν_1, ν_2 and a mixing angle α,

$$\begin{aligned} \nu_e &= \nu_1 \cos\alpha + \nu_2 \sin\alpha \\ \nu_\mu &= -\nu_1 \sin\alpha + \nu_2 \cos\alpha \end{aligned}$$

show that the probability that a ν_e produced at time $t = 0$ is found as a ν_μ at time t is:

$$|B(t)|^2 = \sin^2 2\alpha \sin^2((E_2 - E_1)t/2))$$

where E_1 and E_2 are the energies of ν_1 and ν_2 respectively. Assume that the ν_τ does not mix. **[7]**

An experiment sits 750 km away from a source of electron neutrinos with energy $\approx$ 1 GeV. If the mixing is as above and $\alpha = \pi/3$ and the experiment sees 50% muon neutrinos and 50% electron neutrinos, what is the smallest possible mass difference between the electron neutrino and the muon neutrino? You may assume that the mass of the neutrino is much smaller than its energy. **[5]**

4. The Dirac equation may be written

$$H\psi = (\alpha \cdot \mathbf{p} + \beta m)\psi$$

where $\alpha = (\alpha_1, \alpha_2, \alpha_3)$. Obtain a set of relations which $\alpha_1, \alpha_2, \alpha_3$ and β must satisfy. **[3]**

Show that the relations between α_1, α_2 and α_3 are satisfied by

$$\alpha_i = \begin{pmatrix} -\sigma_i & 0 \\ 0 & \sigma_i \end{pmatrix}$$

where the $\sigma_i, i = 1, 2, 3$ are the Pauli spin matrices. Using this, for the case $m = 0$ decompose the Dirac equation into decoupled equations for the two-component spinors ϕ and χ where

$$\psi = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

[4]

Show that if the helicity operator is $\sigma \cdot \mathbf{p}$, the solutions of one equation describe a negative helicity particle and a positive helicity antiparticle, and the solutions of the other describe a positive helicity particle and a negative helicity antiparticle. **[3]**

Draw a Feynman diagram for the decay $\pi^+ \to \mu^+ \nu_\mu$, indicating where the pion form factor plays a role. **[3]**

Explain in detail why this decay dominates over the decay $\pi^+ \to e^+ \nu_e$. **[7]**

5. Show that the Lagrangian density for an electron:

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$

leads to the Dirac equation and its conjugate equation. **[5]**

Show that this Lagrangian density is invariant under the transformation:

$$\psi(x) \to \psi(x)\exp(i\alpha)$$

where α is a constant. **[3]**

By considering a small change in the Lagrangian density, $\delta\mathcal{L}$, for a transformation involving a small α, derive the continuity equation $\partial_\mu j^\mu = 0$ and an expression for j^μ. **[5]**

Show that the Lagrangian density is not invariant under this transformation if α is a function of x^μ. **[2]**

The invariance can be restored by replacing the derivative ∂_μ by the covariant derivative:

$$D_\mu \equiv \partial_\mu - ieA_\mu$$

where e is a constant. What transformation properties must A_μ have to make this work? What is the physical interpretation of A_μ? **[5]**

 END OF PAPER