Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may assume the following data:

Particle name	Approx.Mass
Z	91 GeV
W	80 GeV
γ	0 GeV
gluon	0 GeV
proton	1 GeV
π	0.14 GeV
charm	3 GeV

Mandelstam variables:

For a two-body scattering $A + B \to C + D$, the Mandelstam variables are given by:

$$\begin{aligned} s &= (p_A + p_B)^2 \\ t &= (p_A - p_C)^2 \\ u &= (p_A - p_D)^2 \end{aligned}$$

where p_A is the four-momentum of particle A, and so on.

Pauli spin matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \boldsymbol{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$$

[Part marks]

1. Draw the lowest order Feynman diagrams for the process $e^+e^- \to e^+e^-$. The propagator carries a four-momentum transfer q. For each diagram give an expression for q^2 in terms of the incoming and/or outgoing particle four-momenta. Give expressions for all the propagators for these diagrams in terms of q. Define all the symbols used. [7]

   Neglecting any effects of the spin of the particles, estimate the dependence upon θ of the *amplitudes* for each of the diagrams, where θ is angle between the beamline and the scattered electron. [6]

   The PETRA collider at DESY collided e^+ and e^- with equal and opposite momenta of 22 GeV each. The LEP collider at CERN collided e^+ and e^- with equal and opposite momenta of 45 GeV, and the LEP2 collider collides e^+ and e^- with equal and opposite momenta of 95 GeV each. In each case a detector measures the angle θ of the final state electron with respect to the beam axis.

   At each collider, which of the above diagrams and propagators are important for $\theta \approx 0$ and $\theta \approx 90^0$? [6]

   The effects of spin were neglected in the above discussion. In practice the spin of the propagator would modify the answers. Give an example of a possible Feynman diagram for $e^+e^- \to e^+e^-$ scattering where this is not the case. [1]

2. Describe the principles of operation of a drift chamber and draw a sketch of a typical chamber in use as a central tracking detector in a collider experiment. Show its position relative to the colliding beams, and a diagram of a small part of it showing the wire layout and field lines. How can a drift chamber be used to determine:

   - Particle momentum
   - The primary event vertex
   - Particle type

   In each case describe which quantities must be read out from the drift chamber and how they are used. [9]

   Charm quarks are produced in high energy ep collisions at HERA. Draw a possible Feynman diagram for such a process. [2]

   10% of the charmed particles produced at HERA decay to final states which include an anti-muon, and 10% to finals states which include an positron. Draw Feynman diagrams for these decays. [3]

   The HERA collider runs for a year and produces 50 pb^{-1} of integrated luminosity. 1000 events with charm quarks decaying to positrons are identified. If the efficiency for finding such an event is 5%, what is the cross section for $ep \to$ charm $+X$? [3]

   Describe a way in which the efficiency of 5% might be evaluated, highlighting any potential problems with the method you suggest. [3]

3. In the quark parton model the hadron structure function F_2 is the sum of the quark momentum densities inside the hadron multiplied by the square of their charges. Define what is meant by momentum density. How do measurements of $\int_0^1 F_2(x)dx$ of the proton and neutron provide evidence for the existence of gluons? Define the variable x and say what meaning it has within the quark parton model. [6]

   Sketch the form you would expect for $F_2^{\mathrm{proton}}(x)$ if the proton was:

   - An elementary particle
   - 3 free quarks
   - 3 quarks bound in a potential well
   - 3 quarks bound by QCD.

   [7]

   Draw the leading order diagram(s) for $u\bar{d} \to \mu^+\nu_\mu$. Draw the quark diagram for such an event at the LHC proton-proton collider, and a schematic picture of such an event in a typical LHC detector. Indicate the colliding beam axis, the muon and the neutrino, and any other features of the final state particles. How could such an event be identified? Where does the $\bar{d}$ quark come from? [7]

4. The Dirac equation can be written

$$H\psi = (\boldsymbol{\alpha} \cdot \mathbf{p} + \beta m)\psi$$

where in the Dirac representation, $\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$.

Evaluate the commutator $[H, \mathbf{L}]$ of the orbital angular momentum with the Hamiltonian, where $\mathbf{L} = \mathbf{r} \times \mathbf{p}$. [4]

Evaluate the commutator $[H, \boldsymbol{\Sigma}]$ of the matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}$$

with the Hamiltonian. [4]

Use these results to argue that the eigenvalues of $\boldsymbol{\Sigma} \cdot \mathbf{p}/|\mathbf{p}|$ are the projection of the spin on the direction of the three momentum - that is, the helicity. [4]

Show that for a massless fermion, there are two independent solutions of the Dirac equation, one of which describes a negative helicity particle and a positive helicity antiparticle, and the other describes a positive helicity particle and a negative helicity antiparticle. [5]

In the standard model, left-handed neutrinos and right-handed antineutrinos experience the weak interaction. Right-handed neutrinos and left-handed antineutrinos do not. In addition, in the standard model the neutrino is massless. Briefly summarize the evidence for and against neutrino mass. [3]

5. Write down the Klein-Gordon equation. Show how a decaying free particle solution to the Klein-Gordon equation,

$$\psi = Ne^{-ip\cdot x}e^{-\Gamma t/2}$$

can be thought of as implying an imaginary term subtracted from the square of the mass, in the case that the mass of the particle $M \gg ,$. [5]

Write down the expression for a massive boson propagator. Use this and the solution above to derive the Breit-Wigner formula for a resonance,

$$\sigma \propto \frac{1}{(W-M)^2 + ,^2/4}$$

where W is the centre of mass energy and is close to M. (You may neglect the $q^\mu q^\nu$ term in the propagator). [5]

Sketch the shape of the cross section for $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ in the range $0 < s < (100\ \mathrm{GeV})^2$ and explain the main features. [3]

List the possible final states the Z may decay to at leading order. Describe briefly how these would appear in a detector. How was the number of types of neutrino measured using these decays at the e^+e^- collider LEP, which collided e^+e^- at $\sqrt{s}$ of around 90 GeV? [7]

END OF PAPER