4260

Queen Mary and Westfield College

UNIVERSITY OF LONDON

MSci EXAMINATION

ELECTROMAGNETIC THEORY I PHY-960

22 MAY 1998 10:00

Time allowed: TWO HOURS

Answer **TWO** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

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PHY/960 Electromagnetic Theory I

Time Allowed : 2 hours.

Answer TWO questions only. All questions carry equal marks.

A Formula Sheet is included at the end of the Paper

1 (a) Explain what is meant by a *gauge transformation* of the electromgnetic potentials $\mathbf{A}, \Phi$.

[7 marks]

(b) What are the electric and magnetic fields described by the potentials

$$\mathbf{A} = \frac{E_0}{ck}\sin(kz - \omega t)\hat{\mathbf{x}},$$

$$\Phi = -\frac{q}{4\pi\epsilon_0}\frac{1}{r}?$$

Here $\omega = ck, r^2 = x^2 + y^2 + z^2$.

[2 marks]

(c) Find a gauge transformation which makes the scalar potential Φ vanish. What is the vector potential $\mathbf{A}$ which results from making this gauge transformation?

[4 marks]

(d) A charged particle, charge Q and mass m is released from rest at the point $(1, 0, 0)$ at time $t = 0$. Taking $qQ > 0$ and assuming the motion to be at all times non-relativistic, give a *qualitative* description of the subsequent motion.

[7 marks]

2 (a) What is meant by the statement: *The components A^μ of the potentials for the electromagnetic fields transform as a contravariant 4-vector.*

[6 marks]

(b) Give the equations which express the electric and magnetic fields $\mathbf{E}', \mathbf{B}'$ in the frame K' in terms of the fields $\mathbf{E} = E\hat{\mathbf{y}}, \mathbf{B} = B\hat{\mathbf{z}}$ in the frame K, where K' is obtained from K by the 'standard boost' along the x-axis.

[7 marks]

(c) A charge q is moving in the frame K with velocity $\mathbf{u} = u\hat{\mathbf{x}}$. Express the Lorentz force $\mathbf{F}'$ on the charge q in the frame K' in terms of the fields E, B and the speed u.

[7 marks]

3 (a) The symmetric stress tensor is given by

$$\Theta^{\alpha\beta} = -\frac{1}{\mu_0}[F^{\lambda\alpha}F_\lambda{}^\beta - \frac{1}{4}\eta^{\alpha\beta}F^{\mu\nu}F_{\mu\nu}].$$

In terms of the electric and magnetic fields, $\mathbf{E}$ and $\mathbf{B}$, obtain expressions for the components Θ^{00}, Θ^{0i} and Θ^{ij} (with $i, j \neq 0$), and state their physical significance.

[9 marks]

(b) Derive the conservation law

$$\partial_\alpha\Theta^{\alpha\beta} = j^\lambda F_\lambda{}^\beta \equiv -f^\beta.$$

[7 marks]

(c) Show that taking $\beta = 0$ this equation expresses the conservation of energy.

[4 marks]

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FORMULA SHEET

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}.$$

$$\begin{aligned}\nabla\cdot(\psi\mathbf{a}) &= \mathbf{a}\cdot\nabla\psi + \psi\nabla\cdot\mathbf{a},\\ \nabla\times(\psi\mathbf{a}) &= (\nabla\psi)\times\mathbf{a} + \psi(\nabla\times\mathbf{a}),\\ \nabla\times(\nabla\times\mathbf{a}) &= \nabla(\nabla\cdot\mathbf{a}) - \nabla^2\mathbf{a}.\end{aligned}$$

$$\nabla(\psi(r)) = \mathbf{n}\psi'(r).$$

Maxwell's equations:

$$\nabla\cdot\mathbf{B} = 0, \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t};$$
$$\nabla\cdot\mathbf{D} = \rho, \qquad \nabla\times\mathbf{H} = \mathbf{j} + \frac{\partial\mathbf{D}}{\partial t}.$$

For linear isotropic media:

$$\mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P},$$
$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}.$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}).$$

The "standard boost":

$$\begin{aligned}x'^0 &= \gamma(x^0 - \beta x^1)\\ x'^1 &= \gamma(x^1 - \beta x^0)\\ x'^2 &= x^2\\ x'^3 &= x^3,\end{aligned}$$

where $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$.

$$\begin{aligned}c^2 d\tau^2 &= c^2dt^2 - dx^2 - dy^2 - dz^2\\ &= dx^\alpha \eta_{\alpha\beta} dx^\beta.\end{aligned}$$

$$\eta_{\alpha\beta} = \begin{cases} +1 & \text{if } \alpha=\beta=0\\ \text{-1} & \text{if } \alpha=\beta=1,2,3\\ 0 & \text{if } \alpha\neq\beta\end{cases}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{1}{c}\frac{\partial}{\partial t}, \nabla\right)$$
$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\nabla\right).$$

$$\Box = \partial_\mu\partial^\mu = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2.$$

$$A^\alpha = (A^0, \mathbf{A}) = \left(\frac{1}{c}\Phi, \mathbf{A}\right) \qquad A_\alpha = \left(\frac{1}{c}\Phi, -\mathbf{A}\right).$$

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -\nabla\Phi - \frac{\partial}{\partial t}\mathbf{A}.$$

$$j^\alpha = (j^0, \mathbf{j}) = (c\rho, \mathbf{j}).$$

$$\Box A^\alpha - \partial^\alpha(\partial_\beta A^\beta) = \mu_0 j^\alpha.$$

$$F^{\alpha\beta} = \partial^\alpha A^\beta - \partial^\beta A^\alpha;$$

$$\|F^{\alpha\beta}\| = \begin{pmatrix} 0 & -E^1/c & -E^2/c & -E^3/c \\ E^1/c & 0 & -B^3 & B^2 \\ E^2/c & B^3 & 0 & -B^1 \\ E^3/c & -B^2 & B^1 & 0 \end{pmatrix}.$$

Maxwell's equations again:

$$\partial_\alpha F^{\alpha\beta} = \mu_0 j^\beta;$$

$$\partial_\alpha F_{\beta\gamma} + \partial_\beta F_{\gamma\alpha} + \partial_\gamma F_{\alpha\beta} = 0.$$

End of paper.