# Royal Holloway

## UNIVERSITY OF LONDON

## MSci EXAMINATION

QUANTUM ELECTRODYNAMICS CP4250A

SUMMER 1998

Time Allowed: TWO HOURS

Answer **TWO** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

**TURN OVER WHEN INSTRUCTED**

CP4250A/20

PART MARKS

Throughout this paper the convention $\hbar \equiv 1$ is used.

1. Write down a differential equation obeyed by the propagator $S_F(x' - x)$ for a free relativistic electron. [1]

Show that for $p^2 \neq m^2$ the corresponding momentum space propagator $\tilde{S}_F(p)$ is given by

$$\tilde{S}_F(p) = (\not{p} - m)^{-1}$$ [5]

where the symbols have their usual meanings.

Also, show using the properties of Dirac $\gamma$ matrices that

$$(\not{p} - m)^{-1} = \frac{(\not{p} + m)}{(p^2 - m^2)}$$ [6]

Show that if $\tilde{S}_F(p) = \dfrac{(\not{p} + m)}{(p^2 - m^2 + i\varepsilon)}$ where $\varepsilon$ is a small positive quantity that tends to zero, then $S_F(x' - x)$ contains only positive frequency components when $t' > t$. [8]

2. For an electron scattering from the fixed Coulomb potential of a nucleus of atomic number $Z$ show that the transition probability per unit time from an initial state $i$ to a final state $f$, with $f \neq i$ is given by

$$\frac{8\pi^3 Z^2 \alpha^2}{E_i E_f V^2} \frac{\left|\bar{u}(p_f, s_f)\gamma^0 u(p_i, s_i)\right|^2}{|\underset{\sim}{q}|^4} \delta(E_f - E_i)$$

where electron wave functions are normalised in a box of volume $V$, $\underset{\sim}{q} \equiv \underset{\sim}{p}_f - \underset{\sim}{p}_i$ and the other symbols have their usual meanings. [9]

Justify the interpretation you use for the square of the Dirac delta function $\delta(E_f - E_i)$ [5]

Also show that the differential cross section for this scattering is given by

$$\frac{d\sigma}{d\Omega} = \frac{Z^2 \alpha^2}{|\underset{\sim}{q}|^4} \left|\bar{u}(p_f, s_f)\gamma^0 u(p_i, s_i)\right|^2$$

evaluated at $E_f = E_i$ and $|\underset{\sim}{p}_f| = |\underset{\sim}{p}_i|$. [6]

TURN OVER

3. The neutral Klein-Gordon field has Lagrangian density, $L$, given by

$$L = \frac{1}{2}\left(\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x_\mu} - m^2\phi^2\right)$$

The field can be expanded in the form

$$\phi(x) = \frac{1}{\sqrt{(2\pi)^3 2E_k}}\int d^3k\left[a(k)\, e^{-ik.x} + a^\dagger(k)\, e^{ik.x}\right]$$

Derive expressions for $a(k)$ and $a^\dagger(k)$ in terms of $\phi$. You may assume any orthonormality properties of $e^{\pm ik.x}$ you require. **[8]**

Use the canonical commutation relations of $\phi$ and its conjugate momentum $\pi$ to show that

$$\left[a(k),\ a^\dagger(k')\right] = \delta\left(\underset{\sim}{k} - \underset{\sim}{k}'\right)$$

and

$$\left[a(k),\ a(k')\right] = 0$$ **[8]**

Assuming that the Hamiltonian H for the neutral Klein-Gordon field can be written in the form

$$H = \frac{1}{2}\int d^3k\, E_k\left[a(k)\, a^\dagger(k) + a^\dagger(k)\, a(k)\right]$$

show that the vacuum energy is infinite. **[4]**

**END**