Royal Holloway

UNIVERSITY OF LONDON

MSci EXAMINATION

STATISTICAL MECHANICS CP4210A

SUMMER 1998

Time Allowed: TWO HOURS

Answer **TWO** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

TURN OVER WHEN INSTRUCTED

CP4210A/25

PART MARKS

1. When the mean field theory of phase transitions is applied to the ferromagnetic transition in the absence of a magnetic field, the free energy is given approximately, in the vicinity of the transition, by

$$F = \frac{Nk}{2}(T - T_c)m^2 + \frac{Nk}{12}T_c m^4.$$

a) Explain the structure of this expression and indicate what the various quantities are. **[4]**

b) By sketching the form of this expression, explain how the transition occurs as the temperature is lowered through T_c and discuss the magnitude of the fluctuations at T_c. **[4]**

c) Using the above expression show that in the vicinity of the transition

$$m = \pm\sqrt{\frac{3(T_c - T)}{T_c}}$$

and sketch this behaviour. **[5]**

d) What is the order of this transition? Explain your reasoning. **[3]**

e) Show that below the transition there is a contribution to the entropy given by

$$\frac{3Nk}{2}(T - T_c)$$

and that this leads to a jump in the thermal capacity on cooling through the transition of

$$\Delta C = \frac{3NkT_c}{2}.$$ **[4]**

2. Answer any *three* of the following questions:

a) Discuss the microscopic basis for the *Law of corresponding states*. You should mention the comparison of experimental data with theoretical models, possible deviations from the law, and the connection with universality. **[6²/₃]**

b) What are *critical exponents* as applied to the study of phase transitions? Discuss the assumptions upon which the scaling theory of critical exponents is based. **[6²/₃]**

c) Discuss the elements of Landau's theory of phase transitions. How are the differences between first and second order phase transitions accounted for? **[6²/₃]**

d) What is the *Ising model*? Discuss its applicability to real systems and (qualitatively) the nature of its solutions in 1, 2, and 3 dimensions. **[6²/₃]**

TURN OVER

PART MARKS

3. When considering a system which can exchange both thermal energy and particles with its surroundings, the probability that it is found in a microstate of energy E with N particles is

$$P \propto \exp -(E-\mu N)/kT.$$

a) What do the symbols T and μ represent in this expression, and how are they related to the equilibrium state of the system? **[3]**

b) Show how the *grand partition function* Ξ is related to the constant of proportionality in the probability expression. **[2]**

c) Using the expression for the entropy

$$S = -k\langle \ln P \rangle$$

show that the grand partition function is related to thermodynamic variables by

$$pV = kT \ln \Xi.$$ **[4]**

d) Show that the mean number of particles in the system, n, is given by

$$n = kT \left. \frac{\partial \ln \Xi}{\partial \mu} \right|_{T,V}.$$ **[3]**

e) Outline the steps by which one proceeds from this to the Bose-Einstein and the Fermi-Dirac distribution functions

$$n_i = \frac{1}{\exp(\varepsilon_i - \mu)/kT \pm 1}.$$ **[8]**

END