Royal Holloway

UNIVERSITY OF LONDON

MSci EXAMINATION

ANALYTICAL MECHANICS CP4200A

SUMMER 1998

Time Allowed: TWO HOURS

Answer **TWO** questions only. No credit will be given for attempting a further question.

Each question carries 20 marks. The mark *provisionally allocated* to each section is indicated in the margin.

TURN OVER WHEN INSTRUCTED

CP4200A/30

PART MARKS

1. A particle of mass m moves under gravity on a smooth sphere of radius R.

Show that the Lagrangian can be written

$$L = \frac{1}{2}m\left(R^2\dot{\theta}^2 + R^2\dot{\phi}^2\sin^2\theta\right) - mgR\cos\theta. \qquad [3]$$

Write down Lagrange's equations and show that the particle satisfies an equation of motion of the form

$$R^2\ddot{\theta} - \frac{B\cos\theta}{m^2R^2\sin^3\theta} - gR\sin\theta = 0$$

where B is a constant. [6]

Write down the Hamiltonian for the problem and obtain the above equation of motion using Hamiltonian methods. [7]

The particle is initially projected horizontally with angular velocity $\dot{\phi} = \omega$ when the radius vector from the centre of the sphere to the particle makes an angle $\theta = \pi/6$ with the vertical.
By integrating the equation of motion, show that $R^2\dot{\theta}^2 + 2gR\cos\theta - gR\sqrt{3} = 0$. [4]

2. (a) Using cylindrical co-ordinates (ρ, θ, z), show that geodesics on a circular cylinder $\rho = a$ (a constant) are helices of the form $\theta = bz + c$, where b and c are constants. [8]

(b) Show that the curve of length $2L$ passing through the two points (0,0) and (L,0), such that the area between the curve and the x-axis is a maximum, is a circle. [8]
Show how all the constants (including the Lagrangian multiplier) can be obtained. [4]

3. Using cylindrical co-ordinates (ρ, θ, z), show that the Hamiltonian for a particle of mass m which is constrained to move under gravity on the inside surface of a frictionless bowl whose shape is given by $\rho^2 = z$, may be written in the form

$$H = \frac{1}{2m}\left(\frac{p_\rho^2}{(1+4\rho^2)} + \frac{p_\theta{}^2}{\rho^2}\right) + mg\rho^2 \qquad [5]$$

where the symbols have their usual meaning.

Write down the Hamilton-Jacobi equation and, by separation of variables, obtain the resulting equations of motion. [15]

END