UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:–

M.Sci.

Astronomy 4C17: Galaxy and Cluster Dynamics

COURSE CODE	:	**ASTR4C17**
UNIT VALUE	:	**0.50**
DATE	:	**03–MAY–05**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05–C0086–3–50

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Solar radius	$R_\odot$	=	6.96×10^{8} m
Solar mass	$M_\odot$	=	2.0×10^{30} kg
Solar Luminosity	$L_\odot$	=	3.83×10^{26} J s^{-1}
Parsec	pc	=	3.09×10^{16} m
Gravitational constant	G	=	6.67×10^{-11} N m^{2} kg^{-2}
Distance to Galactic Centre	R_0	=	10 kpc
Sun's orbital circular velocity	v_0	=	250 kms^{-1}
Oort's constants	A	=	15 kms^{-1} kpc^{-1}
	B	=	−10 kms^{-1} kpc^{-1}

The 1-d Poisson Equation in spherical geometry is:

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)$$

1.(a) Show the mass of a cluster of initial mass M_o and radius r_o varies with time due to evaporation as

$$M(t) = M_o \left(1 - \frac{7\lambda t}{2t^o_{rel}}\right)^{\frac{2}{7}}.$$

You must define all terms and you may assume that the energy of the cluster is fixed and that

$$t_{rel} = \frac{\langle v^2 \rangle^{\frac{3}{2}}}{n}$$

in your derivation. [4]

If the relaxation timescale of a cluster is 10^8 years and $\lambda = 0.008$ show that this model of the cluster evaporation rate predicts a finite lifetime for the cluster. Also show that the core of the cluster will undergo gravothermal collapse. [3]

Hence describe, qualitatively, the Core-Halo model of cluster Evolution. [3]

(b) A virialised globular cluster of mass $M = 2 \times 10^6 M_\odot$ and radius 8 pc has a binding energy of -E. How many binaries of separation $20R_\odot$ would contain as much binding energy as this cluster? You may assume that the mass of each star in a binary system is $1M_\odot$. How many binaries of separation $2R_\odot$ would contain as much binding energy as this cluster. [4]

(c) Simple models of the effect of binary systems on cluster evolution were found to show that binaries do indeed reduce the mass loss rate from a cluster. An additional contribution to the mass loss rate of the form

$$a\frac{M_o^{\frac{7}{2}}}{M^{\frac{5}{2}}}e^{\frac{N}{N_o}},$$

where a and N_o are constants and N is the number of binary systems in the cluster, when added to the evaporation mass loss rate that you used in part (a) was found to slow Core Collapse. Using this term derive a new form of the change of cluster mass with time and show (i) that this modification does slow down core collapse and (ii) find the condition for no cluster evaporation. [4]

If $\lambda/a = 10^7$ years, $t^o_{rel} = 10^8$ years and $N_o = 5$ calculate how many binaries are required to ensure no cluster evaporation. [2]

2.(a) Derive the Toomre Criterion for a flat disk? Hence show that this corresponds to a rotational velocity v in a cloud given by

$$\langle v^2 \rangle^{\frac{1}{2}} = \frac{4G\mu}{3^{\frac{1}{2}} B},$$

where μ is the surface density, B is an Oort constant. [Hint: consider the incremental change in force when a slight contraction occurs in a circular cloud that resides in the Galactic Disk]. You may assume the Jeans Length is given by

$$L_J = \frac{\pi \langle v^2 \rangle}{8G\mu}.$$

State all the assumptions that you have made. **[10]**

(b) For an observed flat disk galaxy the radius was measured to be 10 kpc and the surface density was inferred from the mass to light ratio to be $\mu = 4 \times 10^{10} \mathrm{kg\ m^{-2}}$. The rotation law $v(r)$ for the galaxy was deduced to be flat with a root mean square value of $\langle v^2 \rangle^{\frac{1}{2}} = 200\,\mathrm{km\,s^{-1}}$. Show that the inferred value of μ is incorrect. **[2]**

Use the Toomre criterion, the Jeans Length and the Rotation Length to deduce a range of μ values for this galaxy. Comment on the stability of this galaxy. [You may use the Oort $|B|$ constant for the Milky Way.] **[3]**

(c) A theory of primordial disk galaxy dynamics suggests that giant molecular clouds cannot form if $\langle v^2 \rangle^{\frac{1}{2}} \geq 900\,\mathrm{km\,s^{-1}}$. Calculate stable masses for cloud radii of 50, 500 and 5000 pc respectively. You may use the Oort $|B|$ constant for the Milky Way. Comment on your results. **[5]**

3.(a) Derive the general form of the Virial Equation for a relaxed system composed of N point masses. [6]

How is this equation modified if the system is in equilibrium? What are the main properties of a virial system in equilibrium? Define the virial mass of such a system. [4]

(b) Use the Virial theorem to show that in a relaxed stellar cluster the mean square value of the escape velocity is four times the mean square velocity of the stars in this cluster ($\langle v_{esc.}^2 \rangle = 4 \langle v^2 \rangle$). Define all terms and state all assumptions. [4]

The Full-Width Half Maximum of the CaI line observed towards a Globular cluster is 300 km s^{-1}. What is the escape velocity of the cluster? If the cluster size is 10 pc, what is the Virial mass of the cluster? [2]

(c) Show that, if the cluster evolves primarily as a result of stars driving winds that are *just* ejected from such a cluster, then the radius of the cluster R is related to the mass of the cluster M by

$$R \propto \frac{1}{M}$$

You may assume that the cluster remains well-virialised. [4]

4.(a) How are elliptical galaxies thought to be formed? How are the properties of newly formed elliptical galaxies affected by this formation process? [5]

Show that the lower limit on the neutrino mass m_ν is given by

$$m_\nu \geq \left(\frac{\hbar^3}{\sigma G}\right)^{\frac{1}{4}} r^{-\frac{1}{2}},$$

assuming that neutrinos are the dominant contributor to the mass of cold dark matter and that σ is the *rms* velocity of the neutrinos. Discuss how the observed properties of a cluster of galaxies (assumed to be in Virial Equilibrium) can then be used to determine the lower limit of the mass of the neutrino. [5]

(b) A galaxy's rotation curve is measured to have a constant circular velocity $v_c = 400$ km s^{-1} out to a radial distance of 40 kpc from the galactic centre. How much mass is contained within this radius assuming the galactic halo is spherical? [2]

If the mass-to-light ratio is 10 and the measured galactic luminosity is $10^{10} L_\odot$ within $r \leq 40$ kpc calculate the baryonic mass of the galaxy and comment on the relative proportions of baryonic matter and Dark matter. [2]

(c) By approximating the galaxy as an inifinite plane parallel slab of constant density ρ_o show that within the slab at a height z above the centre of the slab the vertical acceleration can be described by an equation of simple harmonic motion. [4]

In the galactic disk the maximum measured velocity of a star perpendicular to the galactic plane is 10 km s^{-1} and its period T is 100 Myr. Calculate the maximum height of the vertical motion and the local density ρ_o. [2]

5.(a) What does the distribution of stars in phase space f($\underline{r}$, $\underline{v}$,t) physically describe? Use Newton's Second Law of Motion to show that the Collisionless Boltzmann Equation of Stellar Dynamics is given by :-

$$0 = \frac{\partial f}{\partial t} + v.(\nabla_r f) - (\nabla_r \phi_p).(\nabla_v f)$$

where ϕ_p is the gravitational potential of the galaxy and the subscripts r and v refer to derivatives with respect to position and velocity respectively. You should state all the assumptions and simplifications used in this derivation. [10]

(b) If the potential of a spherically symmetric stellar system is given by $\phi = \phi_o \, e^{-br^2}$, where b is a constant and r is the radial distance, then use the Poisson Equation to show that this potential implies a radial density distribution of the form

$$\rho(r) = \frac{2b\phi_o e^{-br^2}[2br^2 - 3]}{4\pi G}$$

Comment on the form of this distribution at small radii. [4]

(c) Show that if $f = \phi e^{-cvt}$, where c is a constant and t is the time, the Collisionless Boltmann Equation is satisfied for the one-dimensional potential given in part (b) when the following equality is obeyed:

$$\phi_o e^{-br^2} e^{-cvt} = \frac{-ve^{-cvt}(c + 2br)}{2brct}$$

[6]

END OF PAPER