UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Astronomy 4C17: Galaxy and Cluster Dynamics

COURSE CODE	:	**ASTR4C17**
UNIT VALUE	:	**0.50**
DATE	:	**02-MAY-03**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

03-C0094-3-40

TURN OVER

Answer THREE questions.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

Solar radius	$R_\odot$	=	6.96×10^{8} m
Solar mass	$M_\odot$	=	1.99×10^{30} kg
Solar Luminosity	$L_\odot$	=	3.83×10^{26} J s^{-1}
Parsec	pc	=	3.09×10^{16} m
Gravitational constant	G	=	6.67×10^{-11} N m^2 kg^{-2}
Distance to Galactic Centre	R_0	=	10 kpc
Sun's circular velocity	v_0	=	250 kms^{-1}
Oort's constants	A	=	15 kms^{-1} kpc^{-1}
	B	=	-10 kms^{-1} kpc^{-1}

$$\int_0^\infty x^2 e^{-a^2x^2}\,dx = \frac{\sqrt{\pi}}{4a^3}$$

In spherical geometry:

$$\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)$$

1.(a) What is the Toomre Criterion for a flat disk? Hence show that the mean internal velocity v of the material in the disk is given by

$$\langle v^2 \rangle^{\frac{1}{2}} = \frac{4G\mu}{3^{\frac{1}{2}}B},$$

where μ is the surface density, B is an Oort constant. [Hint: consider the incremental change in force when a slight contraction occurs in a circular cloud that resides in the Galactic Disk]. You may assume the free fall time is given by

$$t_{ff} = \left(\frac{\pi L}{8G\mu}\right)^{\frac{1}{2}}$$

and the Jeans Length is given by

$$L_J = \frac{\pi \langle v^2 \rangle}{8G\mu},$$

where L is the radius of a circular region that resides in the disk. State all the assumptions that you have made. [8]

(b) For a 10 pc cloud the surface density is given by

$$\mu = \mu_o exp[-r/r_o].$$

Calculate the free fall time assuming $r = r_o$ and $r = 100r_o$, where $\mu_o = 4 \times 10^{10}$kg m^{-2}. Comment on your results. [4]

(c) A theory of primordial disk galaxy dynamics suggests that giant molecular clouds cannot form if $\langle v^2 \rangle^{\frac{1}{2}} \geq 1000\,\mathrm{kms}^{-1}$. Show that the maximum mass M of a giant molecular cloud is linearly related to the the local Oort B constant. Thus calculate stable masses for cloud radii of 10, 100 and 1000 pc respectively. You may use the Oort $|B|$ constant for the Milky Way. [5]

(d) If $\langle v^2 \rangle^{\frac{1}{2}}$ was suddenly damped down to zero which of the 3 molecular clouds described in (c) will collapse first to form stars. How do molecules aid the collapse process? [3]

2.(a) Derive Oort's constants A and B in terms of the local Galactic rotation speed and its radial gradient. Use the Oort model of Galactic rotation to show that the radial and tangential velocities of a star are given by $v_{rad} = Adsin2l$ and $v_{tan} = Adcos2l + Bd$ respectively. State all assumptions and define all terms. [7]

You will need to use the following definitions of Oorts Constants:

$$A + B = -\left(\frac{dv_c}{d\varpi}\right)_{R_o} \quad \text{and} \quad A - B = \frac{v_c}{R_o},$$

and note that

$$\left(\frac{d\dot{\theta}}{d\varpi}\right)_{R_o} = \frac{1}{R_o}\left(\frac{\Theta}{d\varpi}\right)_{R_o} - \frac{\Theta_o}{R_o}.$$

(b) Evaluate the radial dependence of the circular speed of stars, $v_c(\varpi)$, in galaxies whose mass distribution is represented by (i) a point mass M_p, (ii) a homogeneous sphere of density ρ, and (iii) an inhomogeneous sphere in which the density distribution $\rho(\varpi) = k\varpi^{-2}$, where k is a constant. [3]

(c) For each case in part (b) derive the Oort constants at $\varpi = R_o$. [5]

(d) A Schmidt model of our Galaxy gives the rotational velocity in the Galactic plane as $v_c^2(\varpi) = 3.493 \times 10^{-10} P_{-1} \varpi \, \mathrm{m^2 s^{-2}}$ using a density distribution approximated by $\rho(a) = P_{-1}/a$. Use the expression for the rotational velocity given above and the $A - B$ definition of the Oort constants given above in part (a) to obtain P_{-1}. Is this value of P_{-1} physical? Hence state an expression for $\rho(a)$ (in S.I. units). [5]

3.(a) State the general form of the Virial Theorem. What are the main properties of a system in equilibrium. [5]

Use the Virial Theorem to show that the rate at which a galactic cluster loses energy is related to the rate at which its radius changes by the expression

$$\frac{dE}{dt} = \frac{\alpha}{2}\frac{Gm_{cl}^2}{r_{cl}^2}\frac{dr_{cl}}{dt}.$$

You may assume that the gravitational potential energy of a galactic cluster is given by

$$\Omega = \frac{\alpha G m_{cl}^2}{r_{cl}}$$

where m_{cl} is the mass of the cluster, r_{cl} is the radial size of the cluster and α is a geometrical constant. [2]

Assuming constant density ρ_{cl} in the cluster show that

$$\frac{dE}{dt} \propto r_{cl}^2 \frac{d\rho_{cl}}{dt}.$$

[2]

(b) Empirical simulations suggest that the energy E of a cluster evolves as $E = E_o exp[-t]$. If this is the case show by using the appropriate expression for dE/dt that E is a function of r_{cl}^{-1}. Will r_{cl} increase or decrease with time? What happens to the energy and gravitational state of the cluster as a result? [5]

(c) Simulations suggest that the radius of a cluster r_{cl} can be fitted by the following empirical radial law: $r_{cl} = r_o t^{-1}$, where r_o is a constant. Show how the energy of the cluster changes with time for this radial law. What will be the energy and gravitational state of the cluster at $t = 0$? [4]

(d) Comment on the physicality of the $r_{cl} = r_o t^{-1}$ law at late times. [2]

4.(a) What does the distribution of stars in phase space f($\underline{r}, \underline{v}$,t) physically describe? The Collisionless Boltzmann Equation is given by :-

$$0 = \frac{\partial f}{\partial t} + \overline{v}.\nabla_r f - \nabla_r V.\nabla_v f$$

where V is the gravitational potential and the subscripts r and v refer to derivatives with respect to position and velocity respectively. State the physical significance of each term in the Collisionless Boltzmann Equation when applied to the distribution of stars in phase space. **[3]**

Write down the equations that describe the relationships (i) between the density of stars, n, and the distribution function, f and (ii) the gravitational potential of a self-gravitating system and its volume density distribution. **[1]**

Write down the solution to the Collisionless Boltzmann Equation in terms of cylindrical co-ordinates. For a system in steady state that is flat (z=0) and axi-symmetric write down the isolating integrals that describe this system and explain what each isolating integral physically represents. **[4]**

(b) A simple ('isothermal sphere') model of *spherical* globular clusters assumes $f = ke^{-BE}$, where E is the total energy of a star ($E = v^2/2 + \mathrm{V}(r)$), and k and B ($= 3/\langle v^2 \rangle$) are constants. Show that the number density of stars is given by:

$$n(r) = k\left(\frac{2\pi}{B}\right)^{3/2} e^{-B\mathrm{V}(r)}$$

[3]

(c) Using the result for $n(r)$ derived in part (b) write down $n(r)$ for (i) $V(r) = r$ and (ii) $V(r) = \ln r/B$. For each function of $V(r)$ calculate the radius at which $n(r)$ is a minimum. **[6]**

(d) What general, spherically symmetric density distribution $\rho(r)$ is consistent with (i) $V(r) = r$ and (ii) $V(r) = \ln r/B$. Comment on the physicality of each density law. **[3]**

5.(a) Derive an expression for the mass evaporation rate for a virialised globular cluster for which evolution is self-similar. You may assume that the relaxation timescale

$$t_{rel} \propto \frac{\langle v^2 \rangle^{\frac{3}{2}}}{n},$$

where $\langle v^2 \rangle$ is the square of the mean stellar velocity in the cluster, and n is the number density of stars in the cluster. [3]

Write down how both the cluster radius and the number density of stars in the cluster vary with cluster mass. Hence show how the core of the globular cluster will evolve. What happens to the cluster evaporation rate? [2]

Why is a halo also formed? [1]

What is the predicted eventual fate of the core and is this supported by observations? [1]

(b) What causes the mass segregation process in globular clusters? If most stars have $M > 2M_{\odot}$ how is this process affected? [2]

(c) Describe how binaries can affect the evolution of globular clusters? [7]

(d) A virialised globular cluster has a mass M of $10^5 M_{\odot}$ and a radius of $6.8pc$. How many binaries of separation $2R_{\odot}$ would contain as much binding energy as this cluster? You may assume that the mass of each star in a binary system is $1M_{\odot}$. [4]

END OF PAPER