UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Astronomy 4C16: Advanced Topics in Stellar Atmospheres and Evolution

COURSE CODE	:	**ASTR4C16**
UNIT VALUE	:	**0.50**
DATE	:	**04-MAY-05**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05-C0085-3-40

TURN OVER

Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

Equation of Mass Continuity:

$$\dot{M}(r) = 4\pi r^2 \rho v(r)$$

Integration by parts formula:

$$\int U dv = UV - \int V du$$

Solar Luminosity $L_{\odot} = 3.83 \times 10^{26} W$

Detailed Balance Relationship for collisional rate coefficients:

$$\frac{C_{12}}{C_{21}} = \frac{g_2}{g_1} \exp\left(-\frac{h\nu}{kT}\right)$$

Einstein Relations:

$$A_{21} = \frac{2hv^3}{c^2} B_{21} \qquad g_1 B_{12} = g_2 B_{21}$$

Line source functions for a 2 level atom and the general case:

$$S_L = \frac{A_{21}}{B_{21}} / \left[\frac{n_1}{n_2}\frac{B_{12}}{B_{21}} - 1\right] \qquad S_L = \frac{n_j A_{ji}}{(n_i B_{ij} - n_j B_{ji})}$$

Planck Function:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp(\frac{hv}{kT}) - 1\right)^{-1}$$

The complementary function of a 2nd order differential equation:

$$\frac{d^2y}{d^2x} - ax = b \text{ is of the form } y = Ae^{\alpha x} + Be^{\gamma x}$$

Equation of Radiative Transfer

$$I = I_O e^{-\tau_{total}} + \int_0^{\tau_{total}} S e^{-\tau'} d\tau'$$

1. a) Describe, qualitatively, the physical processes that operate in line-driven stellar winds. Which lines, and under what physical conditions, play a major role in driving the winds? [8]

Show that the upper limit for the mass loss rate $\dot{M}(r)$ at a distance r from the stellar centre is given by

$$\dot{M}(r) = \frac{L_{star}}{v(r)\,c}$$

Define all terms and state all assumptions. [2]

b) If

$$\log_{10}\dot{M} = \alpha \log_{10} L_{star} + \log_{10} C$$

derive an expression that gives the total mass lost M_{lost} in a time t assuming L_{star} is constant. [2]

Thus calculate the mass lost in 1000 years from a star with a luminosity of $10^3 L_{\odot}$, $C = 10^{-24}$ and $\alpha = 1.6$. [1]

c) Show that the total mass lost M_{lost} in a time t is given by

$$M_{lost} = M_{initial} - [(2\alpha - 1)C\beta^{\alpha}t + M_{initial}^{1-2\alpha}]^{\frac{1}{1-2\alpha}}$$

$$\text{if } L_{star} = \beta M_{star}^2,$$

where β is a constant and

$$M_{star} = M_{initial} - M_{lost}.$$

[Hint: $dM_{star} = -dM_{lost}$] [7]

2. a) The Lambda iteration scheme that is used to calculate the temperature structure in LTE stellar atmospheres assumes that radiative equilibrium is not satisfied because the temperature used to evaluate the local Planck function is incorrect. Starting with the general equation of radiative equilibrium

$$\int_0^\infty \chi_\nu \, J_\nu \, d\nu = \int_0^\infty \chi_\nu \, S_\nu \, d\nu$$

and assuming LTE, show that the expression for the temperature correction term $\delta T(\tau_0)$ is:

$$\delta T(\tau_0) = \frac{\int_0^\infty \chi_\nu \, [J_\nu(\tau_0) - B_\nu(\tau_0)] \, d\nu}{\int_0^\infty \chi_\nu \left[\frac{\partial B_\nu(\tau_0)}{\partial T}\right] d\nu}$$

[6]

What happens to the temperature correction term derived above at (i) high optical depth and (ii) at low optical depth? [4]

b) Using the equation derived for $\delta T(\tau_o)$ in (a) and assuming that $\chi_\nu = 2$, $B_\nu\,[T_0(\tau_0)] = exp[-2T\nu]$ and $J_\nu(\tau_0) = 0.99B_\nu$ show that $\delta T(\tau_o) = -10^{-2}T$. [5]

c) If $\chi_\nu = \nu A$, $B_\nu\,[T_0(\tau_0)] = exp[-T^2\nu]$ and $J_\nu(\tau_0) = 0.99B_\nu$ determine $\delta T(\tau_o)$. Comment on the relative values of $\delta T(\tau_o)$ calculated in parts (b) and (c) [5]

3. a) Derive the Milne Eddington equation for radiative transfer as given by

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \mathcal{L}_\nu B_\nu - (1 - \mathcal{L}_\nu)\, J_\nu$$

where

$$\mathcal{L}_\nu = \frac{1 + \beta_\nu \epsilon_\nu}{1 + \beta_\nu}$$

and β_ν, a measure of the line strength, is the ratio of the line to continuum opacities, and ϵ_ν is the ratio of the pure absorption opacity to the total opacity for line processes. [7]

State the one-dimensional integral forms of the *three* moments of the radiation field, F_ν, J_ν and K_ν in terms of the intensity I_ν. [3]

b) Thus by taking moments of the Milne-Eddington equation and assuming that $K_\nu = J_\nu/3$ show that

$$\frac{d^2 J_\nu}{d^2\tau_\nu} = 3L_\nu(J_\nu - B_\nu)$$

[4]

c) Using the substitution $B_\nu = Ae^{-\tau_\nu}$ where A is a constant and assuming the Eddington Approximation at the surface $J_\nu(0) = I_\nu(0)/2$, derive the complementary function for the expression

$$\frac{d^2 J_\nu}{d^2\tau_\nu} = 3L_\nu(J_\nu - B_\nu)$$

to determine the general form of J_ν. [You may assume the particular integral is zero] [5]

What are the values of J_ν at high τ_ν and when $\tau_\nu = 1$? [1]

4. a) For a two-level atom, with energy levels denoted by 1 and 2, show that if only bound-bound radiative and collisional processes are considered for the populating and de-populating of the levels, the non-LTE line source function is given by the expression: [10]

$$S_l = \frac{\overline{J} + \epsilon' B_\nu}{1+\epsilon'} = (1-\epsilon)\overline{J} + \epsilon B_\nu$$

where

$$\epsilon = \frac{\epsilon'}{1+\epsilon'}; \quad \epsilon' = \frac{C_{21}}{A_{21}}[1 - \exp(-h\nu/kT)] \text{ and } \overline{J} = \int_0^\infty J_\nu \phi_\nu \, d\nu.$$

b) In a stellar atmosphere of constant temperature T_o, optical depth τ_o and where $J = A\tau$, show that the intensity I at the stellar surface is equal to

$$I = I_o e^{-\tau_o} + \epsilon B_\nu \left(1 - e^{-\tau_o}\right) + (1-\epsilon)\, A \left(1 - e^{-\tau_o} - \tau_o e^{-\tau_o}\right),$$

where I_o is the intensity at optical depth τ_o. [You may assume B_ν and ϵ are constants]. [5]

c) If $B = C\tau$ then what is now the intensity I at the stellar surface? [You may assume ϵ is a constant]. [3]

What are the intensities generated in parts (b) and (c) in both the high and low optical depth limit? Comment on your results. [2]

5. a) Describe qualitatively how P Cygni profiles are formed in a stellar wind. [5]

How can velocity information be determined from a P Cygni profile? State what type of P Cygni profile is used to determine velocity information [3]

How can the mass loss rate be determined from a P Cygni profile? State what type of P Cygni profile is used to determine the mass loss rate [2]

b) If the radio free-free opacity in a completely ionised and isothermal stellar wind is given by $\kappa_\nu = 10^{32}\,\nu^{-1}\,T_w^{-2}\,n_i\,n_e$ m^{-1} then show that the radius r in SI units at which the optical depth τ of the wind is equal to unity is given by

$$r = 5.95 \times 10^9\,\nu^{-1/3}\,T_w^{-2/3}\left(\frac{\dot{M}}{v_\infty}\right)^{2/3} \text{ metres}$$

[5]

Define all terms and list all assumptions

c) Thus show that the mass loss rate for this stellar wind is given by

$$\dot{M} = 2.61 \times 10^{15}\,\nu^{-1}\,T_w^{\frac{1}{4}}\,v_\infty d^{\frac{3}{2}}\,f_\nu^{\frac{3}{4}},$$

where f_ν is the observed radio flux of the wind and d is the distance to the star. [Hint: You may use the Rayleigh-Jeans approximation $B_\nu = 2kT\nu^2/c^2$ in your derivation of the mass loss rate.] [4]

Calculate the mass loss rate for this wind if $\nu = 10^{10}$Hz, $T_w = 10^4$K, $v_\infty =$ 2000km s $^{-1}$, d=100pc and $f_\nu = 10^{-28} W m^{-2}$. [1]