UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Astronomy 4C15: High Energy Astrophysics

COURSE CODE	:	**ASTR4C15**
UNIT VALUE	:	**0.50**
DATE	:	**28-APR-06**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

06-C0093-3-40

TURN OVER

Answer THREE questions.

The numbers in square brackets indicate the provisional allocation of maximum marks per sub-section of a question

Symbols and quantities used in expressions:

Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Gravitational constant	$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Speed of light	$c = 3 \times 10^8$ m s^{-1}
Electron charge	$e = 1.6 \times 10^{-19}$ Coulomb
Magnetic permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
Solar mass	$M_\odot = 2 \times 10^{30}$ kg
Solar radius	$R_\odot = 7 \times 10^8$ m
3 K background photon energy	$h\nu = 3 \times 10^{-4}$ eV
Parsec	pc $= 3.08 \times 10^{16}$ m
Electron volt	eV $= 1.6 \times 10^{-19}$ J

Symbols and quantities used in expressions (in c.g.s. Gaussian units):

Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Boltzmann constant	$k = 1.38 \times 10^{-16}$ erg K^{-1}
Gravitational constant	$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
Reduced Planck constant	$\hbar = 1.0544 \times 10^{-27}$ erg s
Speed of light	$c = 3 \times 10^{10}$ cm s^{-1}
Electron charge	$e = 4.803 \times 10^{-10}$ esu
Electron mass	$m_e = 9.11 \times 10^{-28}$ g (= 511 keV)
Thomson cross section	$\sigma_T = 6.65 \times 10^{-25}$ cm^2
Solar mass	$M_\odot = 2 \times 10^{33}$ g
Parsec	pc $= 3.08 \times 10^{18}$ cm
Electron volt	eV $= 1.6 \times 10^{-12}$ erg

PLEASE TURN OVER

1. Consider a star with a mass $M_* = 2.0 \times 10^{31}$ kg (2.0×10^{34} g) and a radius $R_* = 7.0 \times 10^{9}$ m (7.0×10^{11} cm) and a mean surface magnetic field $B_* = 0.01$ T (0.1 kG). Suppose that the magnetic field strength in the vicinity of the star has a power-law profile $B(r) = B_*(R_*/r)^{3.5}$.

(a) The energy stored in a magnetic field is given by (in c.g.s. Gaussian unit)

$$E = \frac{1}{8\pi} \int d^3x \; |B|^2 .$$

What is the energy of the star's magnetic field? **[5 marks]**

(b) If the star explodes as a supernova and 85% of its mass is ejected, leaving the rest of its mass to collapse and form a compact star with a radius of 10^4 m (10^6 cm), what is the mean density of the compact star? **[2 marks]**

(c) Assume magnetic flux conservation, what is the maximum surface field strength of the compact star? **[2 marks]**

(d) What is the energy stored in the magnetic field of the newly formed compact star? What is the difference between the field energy before and after the supernova/collapse event? **[4 marks]**

(e) The binding energy of a self-gravitating system of mass M and liner size R is $E_g \sim -G M^2 / R$. Show that the collapse can supply sufficient energy to account for the difference in the magnetic field energy before and after the collapse event. **[4 marks]**

(f) If the star collapses to form a black hole of mass 1.0×10^{31} kg (1.0×10^{34} g), what is the magnetic field strength of the black hole (seen by a distant observer)? Justify your answer. **[3 marks]**

CONTINUED

2. (a) State the microscopic and the macroscopic forms of the Maxwell Equations. **[6 marks]**

(b) Consider the microscopic form of the Maxwell Equations. Consider a scalar potential ϕ and a vector potential $\vec{A}$ such that

$$\vec{B} = \nabla \times \vec{A}, \text{ and}$$

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial}{\partial t}\vec{A} \ .$$

Show that the two equations above satisfy the homogeneous pair of the Maxwell Equations. Show that the two inhomogeneous Maxwell Equations can be expressed as

$$\nabla^2\phi + \frac{1}{c}\frac{\partial}{\partial t}\nabla \cdot \vec{A} = -4\pi\rho, \text{ and}$$

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{A} - \nabla\left(\nabla \cdot \vec{A} + \frac{1}{c}\frac{\partial}{\partial t}\phi\right) = -\frac{4\pi}{c}\vec{J} \ .$$

[5 marks]

(c) What is the Lorentz condition? Using the Lorentz condition, show that ϕ and $\vec{A}$ satisfy the inhomogeneous wave equations. **[4 marks]**

(d) Consider the transformations

$$\phi' = \phi - \frac{1}{c}\frac{\partial}{\partial t}\lambda \ , \text{ and}$$

$$\vec{A}' = \vec{A} + \nabla\lambda \ .$$

Show that if the Lorentz condition is preserved, the variable λ satisfies the homogeneous wave equation. **[5 marks]**

PLEASE TURN OVER

3. The formation of a photon spectrum of Comptonized emission from a hot thermal electron cloud of temperature T, electron number density N_e and linear size R can be modeled by the Kompaneet Equation:

$$\frac{\partial}{\partial t}n = \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left[\nu^4 \frac{N_e \sigma_T kT}{m_e c}\frac{\partial}{\partial \nu}n\right] - \frac{nc}{(1+\tau)R} + \sum \lambda_i \delta(\nu - \nu_i),$$

where n is the photon occupation number, ν_i is the frequency of the seed photons, λ_i is the variable that determines the flux of seed photons with frequency ν_i, c is the speed of light, k is the Boltzmann constant, m_e is the electron mass, σ_T is the Thomson cross section, and $\delta(....)$ is the delta function. The intensity of emission is given by

$$dI_\nu = 4\pi\nu^2 n(h\nu)d\nu,$$

where h is the Planck constant, and the scattering optical depth is $\tau = N_e \sigma_T R$.

(a) Show that for $\nu > \nu_i$, in a stationary condition, the Kompaneet Equation above becomes

$$\frac{\theta}{\nu^2}\frac{\partial}{\partial \nu}\left[\nu^4 \frac{\partial}{\partial \nu}n\right] - \frac{n}{\tau(1+\tau)} = 0,$$

where $\theta = kT/m_e c^2$. **[5 marks]**

(b) Assume the spectrum has a power-law profile with spectra index α, and can be expressed as

$$n = n_a \left(\frac{\nu}{\nu_a}\right)^{-(3+\alpha)},$$

where n_a and ν_a are normalization constants. Show that the solution to the stationary Kompaneet Equation is $\alpha(\alpha+3) = 4/y$, where $y = 4\theta\tau(1+\tau)$. **[3 marks]**

(c) Many accreting stellar mass black holes show power-law X-ray spectra in the with spectral indices $\alpha = 0.7$ in the 2 – 10 keV range. It is believed that the emission is due to inverse Compton scattering by hot electron clouds. If the electron number density of the clouds is $N_e \sim 10^{18}\,m^{-3}\,(10^{12}\,cm^{-3})$ and the scattering optical depth $\tau \sim 10^{-1}$, estimate the linear size and the thermal temperature (in K and in keV) of the clouds. **[3 marks]**

(d) If the optical depth is 0.3, what is the corresponding cloud temperature? What are the temperatures for optical depths of 0.5, 1 and 3? (Express the temperatures in keV.) Assume a spectral index α = 0.7 **[2 marks]**

(e) Plot log kT against log τ in the optical depth range from 0.1 to 3 for $\alpha = 0.7$. Based on this plot, generate another one for α = 1.2. Describe qualitatively how the deduced temperature depends on the optical depth. **[7 marks]**

4. a) In order of increasing photon energy, name the four physical processes that result in absorption of X-rays and Gamma-rays by matter. State also the corresponding emission processes.

[4 marks]

b) For interactions with solid matter, show by means of simple plots how the photon absorption cross section varies with energy for three of the above processes. Indicate the approximate energy range in which each process is dominant.

[3 marks]

c) Describe the process of electron-positron pair production.

[3 marks]

d) Show that $E^2 - (pc)^2$ is a relativistic invariant and hence derive an expression for the minimum photon energy required to produce an electron-positron pair.

[7 marks]

e) If the 3 K microwave radiation has an energy density in space of $U_{ph} = 5 \times 10^{-14}$ Jm^{-3}, calculate the corresponding photon number density. Why does the value of this number suggest that high energy Cosmic Ray photons are more likely to interact with other photons than with nuclei?

[3 marks]

PLEASE TURN OVER

5. a) What are Cosmic Rays? Mention the features that give the cosmic radiation astrophysical significance.

[3 marks]

b) What is the difference between primary and secondary Cosmic Rays? Explain how the secondary particles are produced and outline briefly how both types may be detected.

[5 marks]

c) Features of observational interest relating to Cosmic Rays include

- Chemical Composition
- Energy Spectra

Comment briefly on each of these aspects using sketches where appropriate.

[5 marks]

d) In the case of Cosmic Ray particles, how does their interaction with magnetic fields influence our ability to determine their origin? Briefly discuss the isotropy of Cosmic Ray particles.

[4 marks]

e) For protons of energy i) 10^{14} eV and ii) 10^{20} eV, calculate the maximum distance at which it would be possible to determine their sources of origin by direct observation assuming that they travel in a galactic magnetic field of 10^{-11} Tesla. What information can we gain about their origins from the results of your calculation?

[3 marks]

END OF PAPER