UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

B.Sc. *M.Sci.*

Astronomy 2B16: Microscopic Foundations of Astrophysics

COURSE CODE	:	**ASTR2B16**
UNIT VALUE	:	**0.50**
DATE	:	**09-MAY-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours 30 minutes**

02-C0080-3-70

TURN OVER

Answer SIX questions from Section A, and THREE questions from Section B

The numbers in square brackets in the right-hand margin indicate the provisional allocation of maximum marks per sub-section of a question.

You may assume the following (using standard notation):

Mass of the hydrogen atom:	m_H	=	1.67×10^{-27} kg
Mass of the electron:	m_e	=	9.11×10^{-31} kg
Boltzmann's constant:	k	=	1.38×10^{-23} J K^{-1}
Planck's constant:	h	=	6.63×10^{-34} J s
Gravitational constant:	G	=	6.67×10^{-11} N m^2 kg^{-2}
Speed of light:	c	=	3.00×10^8 m s^{-1}
Gas constant:	R	=	8.31 J mol^{-1} K^{-1}
Stefan-Boltzmann constant:	σ	=	5.67×10^{-8} W m^{-2} K^{-4}
Solar luminosity:	$L_\odot$	=	3.90×10^{26} W
Solar radius:	$R_\odot$	=	6.96×10^8 m
Astronomical unit:	(AU)	=	1.496×10^{11} m

SECTION A

1. Explain what is meant by

   (a) an *equation of state*

   (b) a *function of state*. [2]

Which of the following are functions of state: *temperature*, *heat*, *work done*, *entropy*? [2]

Define the *Helmholtz Free Energy*, F, and specify to what type of systems it is most applicable. What happens to F in equilibrium for a system of constant volume in a heat bath? [3]

2. Briefly explain what is meant by the term *heat bath*. [2]

An isolated system is at equilibrium, and is divided into two sub-systems, 1 and 2, separated by a *moveable*, heat-conducting partition. By considering the change in entropy as the partition moves and heat flows from sub-system 1 to sub-system 2, derive the condition that must be satisfied for 1 and 2 to be in equilibrium. [2]

Hence show that this is consistent with the definition of pressure: [3]

$$P = T\left(\frac{\partial S}{\partial V}\right)_{E,N}$$

3. Define the *Einstein coefficients*, A_{ji}, B_{ji} and B_{ij}. [3]

A two-level atom has energy levels that are separated by 1.8×10^{-18}J and with statistical weights of 1 and 6 for the lower and upper levels respectively. Given that $A_{21} = 3\times10^{8}\text{s}^{-1}$, calculate values for B_{21} and B_{12}. [4]

4. State what is meant by the terms *degeneracy* and *Partition Function* in considering the Boltzmann distribution for a single particle system, at constant temperature T, which has possible energy states, E_r. [3]

An atom has three energy levels, with $E_1 = 0.0$ J, $E_2 = 1.38 \times 10^{-19}$ J, and $E_3 = 2.76 \times 10^{-19}$ J, with level degeneracies of 1, 4 and 6 respectively. For a temperature of $T = 10^4$K, calculate the Partition Function of the atom, and the probability that the atom will be its ground state. [4]

5. Explain what is meant by the *microstate* and *macrostate* of a physical system. [2]

State the zeroth, first and second laws of thermodynamics. [4]

What physical property of a system determines whether or not the zeroth law is applicable. [1]

6. Define the terms *absorptivity* and *emissivity* of a body. [2]

Give an expression that relates the stellar luminosity to the effective temperature and the stellar radius. [2]

A star has a luminosity of $25\text{L}_{\odot}$ and an effective temperature of 11,000 K. Using the above expression, calculate the star's radius in units of the solar radius. [3]

7. State the allowed occupation numbers of single particle states for an ideal gas comprised of: [2]

(a) bosons, and

(b) fermions.

Give the <u>two</u> defining properties of a *perfect gas* of atoms and state how the *perfect gas temperature scale* is defined. Define the condition required for a quantal gas to be in the classical regime. Name the statistics that are applicable and write down an expression for the mean number of particles per state at energy E for such systems. [5]

8. What is the definition of a *'black body'*? [1]

Define the following terms used in describing a radiation field, and give the SI units in each case: [3]

(a) radiant flux,

(b) radiant flux density,

(c) radiance and spectral radiance.

The spectral radiance of a black body is given by

$$R_{BB\nu}(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

Use this to derive the *Rayleigh-Jeans* approximation for the spectral radiance of a black body when $h\nu << kT$. [3]

SECTION B

9. In considering the interaction of electromagnetic radiation with atoms, outline what is meant by the process of *scattering* and define the scattering *cross section*. [5]

The mean flux of energy, $\overline{F}$, carried by a plane-polarised electromagnetic wave is given by:

$$\overline{F} = \frac{1}{2}c\epsilon_o E_o^2$$

where c is the velocity of light, ϵ_o is the permeability of free space, and E_0 is the amplitude of the electric field which has the form $E = E_o cos(\omega t)$, with ω being the angular frequency. It can also be shown that a charge, q, undergoing an acceleration $\ddot{l}$, radiates energy per unit time, P, given by:

$$P = \frac{2}{3}\left[\frac{q^2}{4\pi\epsilon_o}\right]\frac{\ddot{l}^2}{c^3}$$

Use these results to show that the *Thomson Cross Section*, σ_T, describing the scattering of electromagnetic radiation by a free electron of mass m is given by [10]

$$\sigma_T = \frac{8\pi}{3}\left[\frac{q^2}{4\pi\epsilon_o mc^2}\right]^2$$

What is the wavelength dependence of electron scattering?
Give an example of an astrophysical environment in which Thomson scattering is particularly important. [2]

What is the wavelength dependence of *Rayleigh Scattering* of electromagnetic radiation, and when will such scattering occur? [3]

10. Explain briefly what is meant by the *escape velocity* of a molecule in the atmosphere of a planet and derive an expression for it in terms of the planet's mass and radius. [6]

For an isothermal planetary atmosphere of temperature T, consisting of a single perfect gas whose molecules have mass m, show that the gas number density, n, as a function of height, h, above the surface can be written as:

$$n = n_o e^{-mgh/kT}$$

where n_o is the surface number density, and it is assumed that the acceleration due to gravity, g, is constant. [8]

A planet has a radius of $R_p = 3 \times 10^6$ m, and a mass of 5×10^{23}kg. If the condition for escape is that the atmospheric scale height is equal to $0.1R_p$ calculate the range of temperature that the isothermal atmosphere can have so that hydrogen, H_2, molecules escape but CO_2 molecules (μ=44) do not. [6]

11. For an ideal gas of N non-interacting Fermions at $T = 0\text{K}$. explain what is meant by the *Fermi energy*, ϵ_F, and the *Fermi temperature*, T_F. [2]

Sketch the Fermi-Dirac distribution for the average ocupation number of single particle states, as a function of energy, at $T = 0\text{K}$. Mark ϵ_F on your diagram. [3]

Consider an ideal Fermi-Dirac gas of N electrons in a volume V at $T = 0\text{K}$. Given that the single-particle density of momentum states is given by;

$$f(p)dp = \left(\frac{V}{h^3}\right) 4\pi p^2 dp$$

show that for this gas the Fermi energy can be written as [8]

$$\epsilon_F = \frac{h^2}{2m_e}\left(\frac{3N}{8\pi V}\right)^{2/3}.$$

Show also that the total average energy is given by [4]

$$\overline{E} = \frac{3}{5}N\epsilon_F.$$

Estimate the Fermi energy *and* the Fermi temperature for electrons in a white dwarf star, for which $N/V = 5.9 \times 10^{36}\text{m}^{-3}$. [3]

12. Give an expression for the Boltzmann distribution of a system in contact with a heat bath at a temperature T, explaining carefully what each term means. [5]

Show that the average energy of a system in contact with a heat bath at a temperature T can be expressed as [5]

$$\overline{E} = -\frac{\partial}{\partial\beta}\ln Z(N, V, T).$$

Use the general definition of entropy,

$$S = -k\sum_j P_j \ln P_j,$$

to show that the entropy of a system in contact with a heat bath at temperature T can be written as [4]

$$S(N, V, T) = k\ln Z(N, V, T) + \overline{E}/T.$$

A system can exist in two quantized energy levels, with energies $E_0 = 2.1 \times 10^{-20}\text{J}$ and $E_1 = 4.2 \times 10^{-20}\text{J}$, and with degeneracies $g_0 = 1$ and $g_1 = 5$ respectively.

If the system is in equilibrium at a temperature of 3000 K calculate: (a) the value of the partition function, (b) the mean energy, (c) the entropy. [6]

13. How does the average (thermal) energy of a gas particle depend on the number of degrees of freedom? [2]

Deduce (with reasoning) the (low-temperature) values of the heat capacity, C_v, for; [6]

(a) a monatomic gas

(b) a gas composed of (diatomic) linear molecules

(c) a gas composed of (triatomic) non-linear molecules.

Why might C_v be different for (b) and (c) in high temperature regions? [2]
The spherically symmetric Maxwellian speed distribution can be written as:

$$F(v)dv = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} dv$$

Explain the meaning of this distribution.
By making use of the substitution $u^2 = mv^2/2kT$, or otherwise, show that the maximum of the distribution occurs at a speed; [8]

$$v_{max.} = \left(\frac{2kT}{m}\right)^{1/2}$$

Calculate $v_{max.}$ for O_2 molecules [$m(O_2)=32m_H$] at T=300K. [2]

END OF PAPER